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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Physics

MM 1131.1 : MATHEMATICS I — DIFFERENTIATION AND ANALYTIC GEOMETRY

(2014 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Find : $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$.

2. State the extreme-value theorem.

3. Evaluate : $\frac{dy}{dx}$ if $xy = 1$.

4. Define a horizontal asymptote of the graph of a function f.

5. Find the rate of change of y with respect to x if $y = -5x + 1$

6. Compute the radius of convergence of the power series $\sum_{k=0}^{\infty} x^k$.

7. State Chain rules for derivatives.

8. Write Euler's theorem for homogeneous functions.

9. State the reflection property of parabolas.

10. Define an ellipse.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Evaluate : $\lim_{x \to +\infty}\left(\frac{1}{x^n}\right)$, if n is a positive integer.

12. Briefly explain a cycloid.

13. State Rolle's theorem.

14. Find the two x - intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point c between those intercepts.

15. Find the local linear approximation of $f(x) = \sin x$ at $x_0 = 0$.

16. Find the Taylor series for $1/x$ about $x = 1$.

17. Let $f(x, y) = x^2y + 5y^3$. Find $f_x\ (1, -2)$ and $f_y = (1, -2)$.

18. Verify Euler's theorem for $f(x, y, z) = x^3 + y^3 + z^3 + 3xyz$.

19. Consider the sphere $x^2 + y^2 + z^2 = 1$. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.

20. Find the rectangular coordinates of the point whose polar coordinates are $(r, \theta) = \left(6, \frac{2\pi}{3}\right)$.

21. How do we sketch a hyperbola from its standard equation?

22. State Kepler's laws.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each carries **4** marks.

23. Use implicit differentiation to find $\frac{d^2y}{dx^2}$ if $4x^2 - 2y^2 = 9$.

24. Use logarithmic differentiation to find $\frac{d}{dx}\left[(x^2+1)^{\sin x}\right]$.

25. Estimate the horizontal asymptotes for $f(x) = \left(1 + \frac{1}{x}\right)^x$.

26. Suppose that x and y are differentiable functions of t and are related by $y = x^3$. Find $\frac{dy}{dt}$ at time $t = 1$ if $x = 2$ and $\frac{dx}{dt} = 4$ at time $t = 1$.

27. Find the second order partial derivatives of $f(x, y) = x^2y^3 + x^4y$.

28. Find the Maclaurin series for

(a) $\cos x$

(b) $\frac{1}{1-x}$.

29. Describe the graph of the equation $16x^2 + 9y^2 - 64x - 54y + 1 = 0$.

30. Sketch the graph of the ellipse $x^2 + 2y^2 = 4$ showing the foci.

31. Find an equation of the parabola that is symmetric about the y - axis, has its vertex at the origin, and passes through the point (5, 2).

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries 15 marks.

32. Suppose that the position function of a particle moving on a coordinate line is given by $s(t) = 2t^3 - 21t^2 + 60t + 3$. Analyse the motion of the particle for $t \geq 0$.

33. An open box is to be made from a 16 – inch by 30 – inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides What size should the squares be to obtain a box with the largest volume?

34. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft^3, and requiring the least amount of material for its construction.

35. (a) Sketch the graph of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ showing their vertices, foci and asymptotes.

(b) Find the equation of the hyperbola with vertices (0, ±8) and asymptotes $y = \pm \frac{4}{3}x$.

(2 × 15 = 30 Marks)