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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course I for Physics

MM 1131.1 – MATHEMATICS I – CALCULUS WITH APPLICATIONS IN PHYSICS I

(2018 – 2019 Admission)

Time : 3 Hours Max. Marks : 80

PART – I

Answer **all** questions. **Each** question carries **1** mark.

1. Find the derivative of $(3+x^2)^3$ with respect to x.

2. State Rolle's Theorem.

3. If $y = f(x)$ and $x = g(t)$ then, $\frac{dy}{dt} =$ ______________.

4. The equation of an Ellipse in polar coordinates with semi-axes a and b is ______________

5. $\int \frac{a}{a^2+x^2}\,dx =$ ______________

6. Sum the integers between 1 and 1000 inclusive.

P.T.O.

7. Determine whether the series $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^n$ converges or not.

8. If $\sum u_n = S$ and $\sum v_n = T$ then $\sum(u_n + v_n) =$ ________.

9. Two particles have velocities $v_1 = i + 3j + 6k$ and $v_2 = i - 2k$ respectively. Find the velocity of the second relative to first particle.

10. Find $|a|$ if $a = 5i - 4j - 7k$.

(10 × 1 = 10 Marks)

PART – II

Answer **any eight** questions. **Each** question carries **2** marks.

11. Find the derivative of $y = a^x$.

12. Use implicit differentiation to find $\frac{dy}{dx}$ if $x^3 - 3xy + y^3 = 2$.

13. Find the derivative of $f(x) = \frac{\sin x}{x}$.

14. Evaluate the integral $\int \ln x \, dx$.

15. Find the mean value of $f(x) = x^2$ between $x = 2$ and $x = 4$.

16. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 2$.

17. Determine whether the series $\sum_{n=1}^{\infty}\frac{1}{n!+1}$ converges.

18. Sum the series $S = 2 + \frac{5}{2} + \frac{8}{2^2} + \frac{11}{2^3} + \ldots$

19. Write the Maclaurin series for (a) $\sin x$ and (b) e^x.

20. Find the direction of the line of intersection of the two planes

$x + 3y - z = 5$ and $2x - 2y + 4z = 3$.

21. Show that if $a = b + \lambda c$, then $a \times c = b \times c$.

22. Is the vector product anti commutative? Justify.

($8 \times 2 = 16$ Marks)

PART – III

Answer **any six** questions. **Each** question carries **4** marks.

23. Find the natures of the stationary points of the function $f(x) = 2x^3 - 3x^2 - 36x + 2$.

24. Find $\frac{dy}{dx}$ if $x = \frac{t-2}{t+2}$ and $y = \frac{2t}{t+1}$.

25. Find the volume of cone enclosed by the surface formed by rotating about the X-axis and the line $y = 2x$ between $x = 0$ and $x = h$.

26. Evaluate the integral $\int \frac{1}{x^2 + 4x + 7} dx$.

27. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+2)}$.

28. Determine the range of value of x for which the power series

    $p(x) = 1 + 2x + 4x^2 + 8x^3 + \ldots$, converges.

29. Find the angle between $a = i + 2j + 3k$ and $b = 2i + 3j + 4k$.

30. Find the area of the parallelogram with side $a = i + 2j + 3k$ and $b = 4i + 5j + 6k$.

31. Find the minimum distance from the point p with $(1,2,1)$ to the line $r = a + \lambda b$, where $a = i + j + k$ and $b = 2i - j + 3k$.

($6 \times 4 = 24$ Marks)

PART – IV

Answer **any two** questions. **Each** question carries **15** marks.

32. (a) Show that the Radius of curvature of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $(3axy)^{\frac{1}{3}}$. **8**

    (b) Show that the curve $x^3 + y^3 - 12x - 8y - 16 = 0$ touches the X-axis. **7**

33. (a) Evaluate the integral $I = \int e^{ax} \cos bx \, dx$. **7**

(b) Using integration by parts find a relation between I_n and I_{n-1} where

$I_n = \int_0^1 (1 - x^3)^n dx$; Hence evaluate $I_2 = \int_0^1 (1 - x^3)^2 dx$. **8**

34. (a) Sum the series $S = 1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots$ **7**

(b) Expand $f(x) = \cos x$ as a Taylor series about $x = \frac{\pi}{3}$. **8**

35. (a) The vertices of triangle ABC have position *vectors a, b, c* from the origin *O*. find the position vector of centroid *G* of the triangle. **10**

(b) Find the volume of the parallelepiped with sides $a = 2i + 3j + k$, $b = i + j + 4k, c = -3i + 2j + 2k$. **5**

(2 × 15 = 30 Marks)
