



Reg. No. : .....

Name : .....

**Third Semester B.Sc. Degree Examination, January 2019**  
**First Degree Programme Under CBCSS**  
**Complementary Course for Physics**  
**MM – 1331.1 : MATHEMATICS – III**  
**Differential Equations, Theory of Equations and Theory of Matrices**  
**(2014 Admission Onwards)**

Time : 3 Hours

Max. Marks : 80

**All the 10 questions are compulsory. Each question carries 1 mark.**

1. Write the 6<sup>th</sup> degree polynomial equation having roots  $\sqrt{2}$ ,  $i\sqrt{2}$  and  $i\sqrt{3}$ .
2. Solve the equation  $x^5 - 16x = 0$ .
3. Find the smallest interval with integer end points containing the positive root of the equation  $x^2 - 3 = 0$ .
4. If  $f(-1) = 1$  and  $f(1) = -1$ , is there a root for the equation  $f(x) = 0$  in  $(-1, 1)$  ?
5. Solve the initial value problem  $\frac{dy}{dt} - y = 0$ ;  $y(0) = \pi$ .
6. Solve  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ .
7. Find the particular integral of  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{5x}$ .
8. Find the rank of the matrix  $\begin{bmatrix} 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$ .
9. Find  $A^3$  if  $A$  is the matrix  $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ .
10. Find the eigenvalues of the matrix  $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ -5 & 0 & 0 \end{bmatrix}$ .



Answer **any 8** questions from among the questions from **11 to 22**. Each question carries **2** marks.

11. Find the condition that the roots of the equation  $x^3 - px^2 + qx - r = 0$  are in arithmetic progression.

12. Solve the equation  $x^3 + x^2 + x + 1 = 0$ .

13. Transform the equation  $x^3 + 3x^2 + 5x + 1 = 0$  into an equation lacking the second term.

14. If  $\alpha, \beta, \gamma$  are the roots of the polynomial equation  $f(x) = 0$ , prove that

$$f'(x) = \left( \frac{1}{x-\alpha} + \frac{1}{x-\beta} + \frac{1}{x-\gamma} \right) f(x).$$

15. Find the general solution of  $e^{xy'} - y^2 = 0$ .

16. Solve  $y' + y - x = 0$ .

17. Solve  $xy' + y = x^4y^4$ .

18. Solve  $y'' + 2y' + 5y = 0$ .

19. Show that inverse of an orthogonal matrix is orthogonal.

20. If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ , show that  $A^2 - 4A + 3I = 0$ .

21. Find the eigenvalues of the matrix  $\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ .

22. Find the rank of the matrix  $\begin{bmatrix} 0 & 1 & 2 \\ 4 & 0 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ .

Answer **any 6** questions from among the questions from **23 to 31**. Each question carries **4** marks.

23. Solve  $x^4 + x^3 + 33x^2 + 61x - 14 = 0$ , given  $2 + \sqrt{3}$  is a root.

24. Using Newton-Raphson method, find a root of the equation  $x^3 + x - 1 = 0$  correct to 6 decimal places ( $x_0 = 1$ ).



25. Using Descartes's rule of signs, show that the equation  $12x^7 - x^4 + 10x^3 - 28 = 0$  has at least four imaginary roots.
26. Find the orthogonal trajectories of the family of parabolas  $y = cx^2$ .
27. Solve  $(y - 2x^3)dx - x(1 - xy)dy = 0$ .
28. Solve  $y'' - 13y + 12y = e^{-2x}$ .
29. Solve  $x^2y' + 2xy - \sin x = 0$ .
30. Test for consistency and solve  $x + y + z = 3$ ,  $x + 2y + 3z = 4$ ,  $x + 4y + 9z = 6$ .
31. Show that the eigenvalues of a real symmetric matrix are real.

Answer **any 2** questions from among the questions from **32 to 35**. **Each** question carries **15** marks.

32. Show that the equation  $x^4 - 10x^3 + 23x^2 - 6x - 15 = 0$  can be transformed into a reciprocal equation by diminishing its roots by 2. Hence solve the given equation.
33. Solve the equation  $x^3 - 3x^2 + 12x + 16 = 0$  using Cardan's method.
34. a) Solve  $y'' + 16y = e^{-3x} - \cos 4x$ .
- b) Solve the initial value problem  $x^2y' - y^2y' - 2xy = 0$ ,  $y(1) = 1$ .
35. Is the matrix  $\begin{bmatrix} 2 & -2 & 1 \\ -1 & 3 & -1 \\ 2 & -4 & 3 \end{bmatrix}$  diagonalizable? If yes, find the diagonal equivalent.
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