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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, October 2019

First Degree Programme Under CBCSS

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III — CALCULUS AND LINEAR ALGEBRA

(2018 admission)

Time : 3 Hours Max. Marks : 80

PART – A

All the **ten** questions are compulsory. They carry **1** mark each .

1. Define an exact first degree first order ODE.

2. Write the general form of Bernoulli's equation.

3. Give an example for a linear first order ODE.

4. When is a vector field said to be conservative?

5. Write the continuity equation.

6. Give the integral form for divergence.

7. Give an example for a periodic function.

8. Define commutator of two matrices A and B.

P.T.O.

9. Find AB if $A=\begin{pmatrix} 3 & -4 \\ -4 & 2 \end{pmatrix}, B=\begin{pmatrix} 5 & 2 \\ -7 & 3 \end{pmatrix}$.

10. Define linear function of vectors.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions from among the questions 11 to 22. These questions carry **2** mark each.

11. Solve the ODE $y'+2\sin 2\pi x=0$ by integration.

12. State Stoke's Theorem.

13. State divergence theorem.

14. Find the period and frequency of the function $s=\frac{1}{2}\cot(\pi t-8)$.

15. Give an example of a function which fails to satisfy the Dirichlet conditions but expandable in a Fourier series.

16. Solve $\frac{dy}{dx}=x+xy$.

17. Verify that $y=ce^{-4x}+0.35$ is a solution of the ODE $y'+4y=1.4$.

18. Give an expression for the angular momentum of a solid body rotating with angular velocity ω about an axis through the origin.

19. Write e^x as the sum of an even function and an odd function.

20. State divergence theorem.

21. Find the direction of the line of intersection of the planes $x-2y+3z=4$ and $2x+y-z=5$.

22. If $\vec{F}=2xz\hat{i}+2yz^2\hat{j}+(x^2+2y^2z-1)\hat{k}$, find $\nabla\times\vec{F}$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions from among the following questions 23 to 31. These questions carry **4** marks each.

23. Solve $x\frac{dy}{dx}+y=xy^3$.

24. Solve $(x+2y^3)\frac{dy}{dx}=y$.

25. Show that the equation

$(1+4xy+2y^2)dx+(1+4xy+2x^2)dy=0$ is exact and solve it.

26. From Ampere's law, derive Maxwell's equation in the case where the currents are steady, ie., $\nabla\times\vec{B}=\mu_0\vec{J}=0$.

27. Find the vector area of the surface of the Hemisphere $x^2+y^2+z^2=a^2, z\geq 0$ by evaluating the line integral $\vec{S}=\frac{1}{2}\oint_C \vec{r}\times d\vec{r}$ around its perimeter.

28. Represent $f(x)=\begin{cases}1, & -1<x<1\\ 0, & |x|>1\end{cases}$ as a Fourier integral.

29. Find the angle between the vectors $\vec{A}=3\hat{i}+6\hat{j}+9\hat{k}$ and $\vec{B}=-2\hat{i}+3\hat{j}+\hat{k}$.

30. Find the rank of the matrix $\begin{pmatrix} 1 & -1 & 2 & 3 \\ -2 & 2 & -1 & 0 \\ 4 & -4 & 5 & 6 \end{pmatrix}$.

31. Find the dimension of the space spanned by the vectors

$\{(1, 0, 1, 5, -2), (0, 1, 0, 6, -3), (2, -1, 2, 4, 1), (3, 0, 3, 15, -6)\}$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions from among the following questions 32 to 35. These questions carry **15** marks each.

32. Use variation of parameters method to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$, subject to the boundary conditions $y(0) = y(\pi/2) = 0$.

33. Given the vector field $\vec{a} = y\hat{i} - x\hat{j} + z\hat{k}$ verify Stoke's theorem for the hemispherical sphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.

34. Find the Fourier series for $|x|$ in $[-\pi, \pi]$ and deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

35. Given the matrices

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 0 & -1 \\ 4 & -2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

(a) Find A^{-1}, B^{-1}, $B^{-1}AB$ and $B^{-1}A^{-1}B$.

(b) Show that the matrices $B^{-1}AB$ and $B^{-1}A^{-1}B$ are inverses.

(2 × 15 = 30 Marks)