(Pages : 6) **G – 2444**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme under *CBCSS*

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II — INTEGRATION AND VECTORS

(2014-2017 Administration)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer the first **ten** questions are compulsory. They carry 1 mark each.

1. Evaluate $\int \tan^2 x \, dx$.

2. Find the position function of a particle that moves with velocity $v(t) = \cos \pi t$ along a coordinate line, assuming that the particle has coordinate s = 4 at time t = 0.

3. Suppose that a particle moves on a coordinate line so that its velocity at time t is $v(t) = |t-3| \ m/s$. Find the displacement of the particle during the time interval $0 \le t \le 5$.

4. Find the average value of the function $f(x) = \sqrt{x}$ over the interval [1,4].

P.T.O.

What do you mean by Surface of revolution?

A particle move along a circular path in such a way that its x and y coordinates at time t are $x = 2\cos t$, $y = 2\sin t$. Find the speed of the particle at time t.

Find the slope of the line in 2-space that is represented by the vector equation $r(\vec{t}) = (1-2t)\,\hat{i} - (2-3t)\,\hat{j}$.

8. Find a unit vector $\vec{u}$ that is normal at P(1, –2) to the level curve of $f(x, y) = 4x^2 y$ through P.

9. Define inverse square field.

10. If $\phi(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ is a potential function of the vector field $\vec{F}(x, y) = x\hat{i} + y\hat{j}$, find $\int_{(0,0)}^{(1,1)} \vec{F} \cdot d\vec{r}$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the questions 11 to 22. These questions carry 2 marks each.

11. Find the equation of the curve that satisfies the following conditions: At each point (x, y) on the curve, y satisfies the condition $\frac{d^2y}{dx^2} = 6x$; the line $y = 5 - 3x$ is a tangent to the curve at the point where $x = 1$.

12. A penny is released from rest near the top of the time Empire State Building at a point that is 1250 ft above the ground. Assuming that the free-fall model applies, how long does it take for the penny to hit the ground, and what is its speed at the time of impact.

13. Find the total area between the curve $y = 1 - x^2$ and the x– axis over the interval [0, 2].

14. A projectile fired downward from a height of 112 *ft* reaches the ground in 2 s. What is its initial velocity.

15. Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval [1, 4] is revolved about the x- axis.

16. If $\vec{r} = x\vec{i} + y\vec{j}$, show that $\nabla r = \frac{\vec{r}}{r}$ where $r = \sqrt{x^2 + y^2}$.

17. Use double integral to find the volume under the surface $z = 3x^3 + 3x^2y$ over the rectangle R = [1, 3] × [0, 2].

18. Use polar coordinates to evaluate $\iint_R \frac{1}{1 + x^2 + y^2} dA$, where R is the sector in the first quadrant bounded by $y = 0$, $y = x$ and $x^2 + y^2 = 4$.

19. A bug, starting at the reference point (1, 0, 0) of the curve $\vec{r} = \cos t\,\hat{i} + \sin t\,\hat{j} + t\hat{k}$ walks up the curve for a distance of 10 units. What are the bug's final coordinates.

20. Find the curvature of the smooth curve $r(\vec{t}) = e^t\,\hat{i} + e^{-t}\,\hat{j} + t\hat{k}$ at the point t = 0.

21. Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at the point (1, –2, 0) in the direction of the vector $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$.

22. Find the divergence and the curl of the vector field $\vec{F}(x, y, z) = x^2y\hat{i} + 2y^3z\,\hat{j} + 3z\hat{k}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions 23 to 31. These questions carry 4 marks each.

23. A car traveling 60 *mi/h* along a straight road decelerates at a constant rate of 10 ft/s^2.

    (a) How long will it take until the speed is 45 *mi/h*

    (b) How far will the car travel before coming to a stop.

24. Sketch the region enclosed by the curves $y = x^2$, $y = \sqrt{x}$, $x = \frac{1}{4}$, $x = 1$ and find its area.

25. Find the flux of the vector field $\vec{F}(x, y, z) = z\hat{k}$ across the outward-oriented sphere $x^2 + y^2 + z^2 = a^2$.

26. Evaluate $\int_1^3 \int_0^{in\,x} x\,dy\,dx$ by changing the order of integration.

27. Use spherical coordinates to find the volume of the solid within the cone $\phi = \frac{\pi}{4}$ and between the spheres ρ = 1 and ρ = 2.

28. Suppose a particle moves along the parabola y = x2 with constant speed oı 3 units per second. Find the normal scalar component of acceleration as a function of x.

29. A heat-seeking particle is located at the point (2, 3) on a flat. metal plane whose temperature at a point (x, y) is T(x, y) = $10 - 8x^2 - 2y^2$. Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

30. Evaluate $\int_C 2xy\,dx + \left(x^2 + y^2\right)dy$ along the circular are C given by

$x = \cos t,\ y = \sin t\,(0 \le t \le \pi/2)$.

31. Find the work done by the force field $\vec{F}(x, y) = x^3 y\,\hat{i} + (x - y)\,\hat{j}$ on a particle that moves along the parabola $y = x^2$ from (–2.4) to (1, 1)

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. These questions carry 15 marks each.

32. (a) Find the area of the region enclosed by $x = y^2$ and $y = x - 2$, integrating with respect to x. (7)

(b) Find the area of the surface generated by revolving the curve, $x = y^3$ between $y = 0$ and $y = 1$ about the y – axis. (8)

33. (a) Evaluate $\iint_G xyz\,dV$ where G is the is the solid in the first octant that is bounded by the parabolic cylinder $z = 2 - x^2$ and the plane s $z = 0$, $y = x$ and $y = 0$. (8)

(b) A bus has stopped to pick up riders, and a woman is running at a constant velocity of 5 m/s to catch it. When she is 11 m behind the front door the bus pulls away with a constant acceleration of 1 m/s^2. From that point in time, how long will it take for the woman to reach the front door of the bus if she keeps running with a velocity of 5 m/s. (7)

34. (a) Find the mass of a thin wire shaped in the form of the circular arc $y = \sqrt{9 - x^2}$ $(0 \le x \le 3)$ if the density function is $\delta(x, y) = x\sqrt{y}$. (7)

(b) Verify Green's theorem for $f(x, y) = y^2, g(x, y) = x^2$ and C is the square with vertices (0,0), (1,0), (1, 1) and (0, 1) oriented counterclockwise. (8)

35. Verify Stokes' Theorem for the vector field $\vec{F}(x, y, z) = 2z\hat{i} + 3x\hat{j} + 5y\hat{k}$, taking a to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \ge 0$ with upward orientation, and C to he the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy— plane.

(2 × 15 = 30 Marks)