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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme under CBCSS

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – INTEGRATION AND VECTORS

(2013 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer all questions. Each question carries 1 mark.

1. Find the average value of $f(x) = \sin x$ over $[0, \pi/2]$.

2. Solve the differential equation $\frac{dy}{dx} = \frac{1}{(2x)^3}$; $y(1) = 0$.

3. Suppose that a particle moves on a co-ordinate line so that its velocity at time t is $v(t) = 2t - 4\ m/s$. Find the displacement of the particle during the time interval $0 \le t \le 6$.

4. Find the area between the curve $y = \sin \pi x$ and the x–axis over the interval $[0, 1]$.

5. Find the path described by the moving particle with position vector $\vec{r}(t) = 3t^2\vec{i} + 2t\vec{j}$.

P.T.O.

6. Find $\vec{r}(t)$, if $\vec{r}(t) = 3\vec{i} + 2t\vec{j}$ and $\vec{r}(1) = 2\vec{i} + 5\vec{j}$.

7. Prove that $\vec{F}(x, y) = x\vec{i} + y\vec{j}$ is a conservative field and find the scalar potential.

8. Find the directional derivative of $f(x, y) = xy^2$ at $P(2, 1)$ in the direction of $\vec{a} = 4\vec{i} - 3\vec{j}$.

9. Use the fundamental theorem of work integrals to evaluate the work done by the force field $\vec{F} = (x, y) = y\vec{i} + x\vec{j}$ on a particle that moves from (0,0) to (1,1).

10. What is meant by an solenoida vector field?

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. Each question carries 2 marks.

11. A stone is thrown downwards with an initial speed of 96 ft/sec from a height of 112ft. Assume that a free fall model applies how long will it take for the stone to hit *the* ground and what is its speed at the time of impact?

12. A particle moves with acceleration -2 m/sec^2 on s-axis and velocity 3 m/sec at t=0. Find the displacement of the particle during the time interval, $1 \le t \le 2$.

13. Find the function f such that $f''(x) = x + \cos x$, $f(0) = 1$, and $f'(0) = 2$.

14. Find the area of the region enclosed between $y = x^2$ and $y = 4x$.

15. Evaluate $\int \frac{3x}{\sqrt{4x^2+5}}\, dx$.

16. Suppose that the velocity function of a particle moving along a co-ordinate line is $v(t) = 6t + 1$. Find the average velocity of the particle over the time interval, $1 \le t \le 5$ by integration.

17. Sketch the line segment represented by the vector equation

$\vec{r}(t) = (1-t)(\vec{i} + \vec{j}) + t(\vec{i} - \vec{j})$; $0 \le t \le 1$

18. Prove that $curl\left(\vec{F}+\vec{G}\right)=curl\ \vec{F}+curl\ \vec{G}$.

19. Prove that $div\,\vec{r}=3$.

20. Verify that the *vector* field $\vec{F}\,(x, y)=e^{y}\vec{i}+x\,e^{y}\,\vec{j}$ is a conservative field in the entire $x-y$ plane.

21. Determine the constant a so that $\vec{F}\,(x, y, z)=2\,(x, y)\,\vec{i}+(y-z)\,\vec{j}+a\,(z-x)\,\vec{k}$ is solenoidal.

22. Use divergence theorem to find the outward flux of the vector field $\vec{F}\,(x, y, z)=2\,x\vec{i}+3\,y\,\vec{j}+z^2\,\vec{k}$ across the unit cube.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries 4 marks.

23. Use double integral to find the area of the region enclosed between the parabola $y=\frac{1}{2}x^2$ and the line $y=2x$.

24. Find the length of the cardioid $r=a\,(1-\cos\theta)$.

25. Find the area of the region that is inside the cardioid $r=4+4\cos\theta$, and outside the circle $r=6$.

26. Find the area of the surface generated by revolving the portion of the curve $y=x^2$ between $x=1$ and $x=2$ about the y-axis.

27. Find the volume of the elliptic cylinder $x^2+9y^2=9$ bounded by the planes $z=0$ and $z=x+3$.

28. Use spherical co-ordinates to find the volume of the solid G bounded above by the sphere $x^2+y^2+z^2=16$ and below by the cone $z=\sqrt{x^2+y^2}$.

29. Prove that $curl\,(grad\,\phi)=0$ where ϕ is a scalar point function.

30. A particle moves in space so that its position vector at time t is $\vec{r}(t) = \sin t\,\vec{i} + \cos t\,\vec{j} + t\vec{k}$. Find the tangential and normal unit vector at $t_0 = 0$.

31. Find the curvature $k(t)$ for the curve $x = 3\cos t$, $y = 4\sin t$, $z = t$ at $t = \pi/2$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

32. (a) Evaluate $\iint_R x\sqrt{1-x^2}\,dA$ over the region, $R = \{(x, y); 0 \le x \le 1, 0 \le y \le 1\}$.

(b) Use a double integral to find the volume of the solid bounded above by the planes $z = 4 - x - y$ and below by the rectangle $0 \le x \le 1$, $0 \le y \le 2$.

33. Find the maximum height, flight time and range of a projectile fired from the origin over horizontal ground at an initial speed of 500 m/sec and a launch angle of 45°. Where will be the projectile 10 seconds later?

34. (a) Evaluate the integral $\oint_C (e^x + y^2)dx + (e^y + x^2)dy$, where C is the boundary of the region between $y = x^2$ and $y = x$.

(b) Use divergence theorem to find the outward flux of the vector field $\vec{F}(x, y, z) = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ across the surface of the region enclosed by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and the plane $z = 0$.

35. Find the work performed by the vector field $\vec{F}(x, y, z) = x^2\vec{i} + 4xy^3\vec{j} + y^2x\vec{k}$ on a particle that traverses the rectangle C in the plane $z = y$, with vertices (0, 0, 0), (1, 0, 0), (0, 3, 3) and (1, 3, 3).

(2 × 15 = 30 Marks)
