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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021.

First Degree Programme under CBCSS

Mathematics

Complementary Course for Phsyics

MM 1231.1 – MATHEMATICS – II – INTEGRATION AND VETORS

(2014–2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. The area $A(x)$ under the graph of f and over the interval $[a,x]$ is x^2-4. Find the function f and the value of a.

2. Evaluate $\int \frac{\sec x}{\cos x} dx$.

3. True or False: "If the particle has constant nonzero acceleration, its position versus time curve will be a parabola."

4. Find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval [1,4] is revolved about the x-axis.

5. Find the limit of $\lim_{t\to 0^+} (ti + \frac{\sin t}{t} j)$.

6. Define inverse-square field.

7. Find the divergence of $F(x,y,z) = x^2yi + 2y^3zj + 3zk$.

P.T.O.

8. Find the ∇f of $f(x,y,z) = xy^2z^3$ at the point (1,1,1).

9. Find the outward ux of $F(x,y,z) = xi + yj + zk$ across any unit cube.

10. State Gauss Divergence Theorem.

SECTION – II

Answer any 8 questions from among the questions 11 to 22

These questions carry 2 marks each

11. Find the area under the curve $f = e^{2x}$ over the interval $[0, \ln 2]$.

12. Find parametric equations of the line tangent to the graph of $r(t) = t^2 i + (2 - \ln t) j$ *at* the point where t = 1.

13. Find the volume of the solid that results when the region enclosed by $y = \sqrt{25 - x^2}$ $y = 3$ and it is revolved about the x-axis.

14. Find the area of the surface generated by revolving the curve $x = 9y + 1, 0 \leq y \leq 2$ about the y-axis.

15. Find $\int_0^{\pi/3} \int_0^{\cos y} x \sin y \, dx \, dy$.

16. Find $\int_1^3 \int_x^{x^2} \int_0^{\ln z} x e^y \, dy \, dz \, dx$.

17. Find a unit vector in the direction in which $f(x,y) - 4x^3y^2$ increases most rapidly at $P = (-1,1)$.

18. Find two unit vectors that are normal to the surface $\sin xz - 4\cos yz = 4$ at the point $P = (\pi, \pi, 1)$

19. Show that $f(x,y,z) = 2z^3 - 3((x^2 + y^2)z)$ satisfies Laplace' Ėquation.

20. Determine whether the statement

    "The line integral of a continuous vector eld along a smooth curve C is a vector". is true or false. Explain your answer

21. Show that the line integral $\int_C y \sin x \, dx - \cos x \, dy$ is independent *of* the path.

22. If σ is the sphere of radius 2 centered at the origin, then find $\iint_\sigma (x^2 + y^2 + z^2) dS$.

SECTION – III

Answer any 6 questions from among the questions *23* to 31

These questions carry 4 marks each

23. Find a positive value of k such that the average value of $f(x)=\sqrt{3x}$ over the interval $[0,k]$ is 6

24. Find the volume of the solid whose base is the region en- closed between the curve $x=1-y^2$ and the $y-$ axis and whose cross sections taken perpendicular to the $y-$axis are squares.

25. Find the volume of the solid that results when the region enclosed by $y=x^2$ and $y=x^3$ is revolved about the line $x=1$.

26. Use a double integral to and the volume of *the* tetrahedron bounded by the coordinate planes and the plane $z=4-4x-2y$.

27. Use a triple integral to and the volume of the solid within the cylinder $x^2+y^2=9$ and between the planes $z=1$ and $x+z=5$.

28. Show that the ellipsoid $2x^2+3y^2+z^2=9$ and the sphere $x^2+y^2+z^2-6x-8y-8z+24=0$ have a common tangent plane at the point (1,1,2).

29. Let k be a constant, $F=F(x,y,z), G=G(x,y,z)$, and $\phi=\phi(x,y,z)$ Show that

    (a) curl (F + G) = curl F + curl G

    (b) $div(\phi F)=\phi\, div F+\nabla\phi.F$

30. Find the area of the surface extending upward from the circle $x^2+y^2=1$ in the $xy-$plane to the parabolic cylinder $z=1-x^2$.

31. Find a nonzero function h for which

    $F(x,y)=h(x)(x\sin y+y\cos y)i+h(x)(x\cos y-y\sin y)j$ is conservative

SECTION – IV

Answer **any two** questions from among the questions 32 to 35

These questions carry 15 marks each

32. (a) Find the area between the curve $y = \sin x$ and the line segment joining the points (0,0) and $(5\pi/6, 1/2)$ on the curve

(b) Let G be the wedge in the rst octant that is cut from the cylindrical solid $y^2 + z^2 \leq 1$ by the planes $y = x$ and $x = 0$. Evaluate.

$$\iiint_G z\,dV$$

33. (a) A shell, red from ground level at an elevation angle of 45° hits the ground 24,500 meter away. Calculate the muzzle speed of the shell.

(b) Given that the directional derivative of $f(x,y,z)$ at the point (3,-2,1) in the direction of $a = 2i - j - 2k$ is -5 *and* that $\|\nabla f(3,-2,1)\| = 5$, find $\nabla f(3,-2,1)$.

34. (a) $F(x,y,z) = z^2 i + 2xj - y^3 k$; C is the circle $x^2 + y^2 = 1$in the xy–plane with counterclockwise orientation looking down the positive z–axis. Use Stoke's theorem to evaluate $\oint_C F.dr$.

(b) Use the Divergence Theorem to and the mc of $F(x,y,z) = x^3 i + y^3 j + z^3 k$ across the surface σ with onward orientation; σ is the surface of the cylindrical solid bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

35. (a) Find the work done by the force field

$$F(x,y) = (e^x - y^0 3)i + (\cos y + x^3)j$$

on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counterclockwise direction.

(b) Evaluate the surface integral $\iint_\sigma f\,dS$ where $d(x,y,z) = x + y : \sigma$; is the portion of plane $z = 6 - 2x - 3y$ in the rst octant.
