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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, August 2018
First Degree Programme Under CBCSS
Complementary Course for Physics
MM 1231.1 : MATHEMATICS – II : Integration and Vectors
(2013 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry **1** mark **each**.

1. Evaluate $\int \frac{t^2 - 2t^4}{t^4} dt$.

2. What is the surface area of the surface of revolution that is generated by revolving the portion of the curve y = f(x) between x = a and x = b about the x-axis ?

3. Evaluate $\int_{-1}^{0} \int_{2}^{5} dx\, dy$.

4. Find the arc length of the curve x = cos 2t, y = sin 2t, $0 \le t \le \frac{\pi}{2}$.

5. Suppose that $\vec{u} \cdot (\vec{v} \times \vec{w}) = 3$, find $(\vec{u} \times \vec{w}) \cdot \vec{v}$.

6. Find $\vec{r}'(t)$ where $\vec{r}(t) = \frac{1}{t} i + \tan t\, j + e^{2t} k$.

7. Find the divergence of the radius vector xi + yj + zk.

8. Let a, b and c be nonzero real numbers such that the vector field $x^5 y^a i + x^b y^c j$ is a conservative field. Find a, b, c.

9. Find $\int_C -y\, dx + x\, dy$, where C is the square with vertices (±1, ±1) oriented counterclockwise.

10. Consider the surface integral $\iint_\sigma f(x, y, z)\, dS$ where σ is the parametric surface whose vector equation is $\vec{r}(u, v) = x(u, v)\, i + y(u, v)\, j + z(u, v)\, k$. What should replace dS to evaluate the integral ?

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Evaluate $\int_{-1}^{2} |2x - 3|\, dx$.

12. Find the arc length of the curve $x = \frac{1}{3}(y^2 + 2)^{3/2}$ from y = 0 to y = 1.

13. Evaluate $\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{1-r^2}} zr\, dz\, dr\, d\theta$.

14. A particle moves with a velocity of v(t) = 2t – 4 m/s along an x-axis. Find the displacement and the distance travelled by the particle during the time interval $0 \le t \le 6$.

15. Use a scalar triple product to determine whether the vectors $\vec{u} = \langle 5, -2, 1 \rangle$, $\vec{v} = \langle 4, -1, 1 \rangle$, $\vec{w} = \langle 1, -1, 0 \rangle$ lie in the same plane.

16. Find the unit tangent vector to the curve $\vec{r}(t) = (t^2 - 1)\, i + t\, j$ at the point t = 1.

17. Verify whether the vector $\vec{F}(x, y, z) = \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}$ is irrotational ?

18. Find the speed and acceleration at time $t = \frac{\pi}{4}$ of a particle moving along the curve x = 2 cost, y = 2 sint, z = t.

19. Evaluate $\int_C yz\, dx - xz\, dy + xy\, dz$ where C is the curve given by $x = e^t$, $y = e^{3t}$, $z = e^{-t}$, $0 \le t \le 1$.

20. Use a line integral to find the area of the region enclosed by the triangle with vertices (0, 0), (a, 0) and (0, b) where a > 0, b > 0.

21. State Gauss's Divergence theorem.

22. Verify whether $\vec{F}(x, y, z) = xyi + x^2j + \sin zk$ is a conservative field.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Evaluate $\iiint_G 12xy^2z^3\ dV$ over the rectangular box G defined by the inequalities $-1 \leq x \leq 2,\ 0 \leq y \leq 3,\ 0 \leq z \leq 2$.

24. Find the area of the region enclosed by the curves $y = x^2$, $y = \sqrt{x}$, $x = \frac{1}{4}$, $x = 1$.

25. Find the volume in the first octant bounded by the coordinate planes, the plane $y = 4$ and the plane $\frac{x}{3} + \frac{z}{5} = 1$.

26. Suppose that a particle moves through 3-space so that its position vector at time t is $\vec{r}(t) = e^t i + e^{-t} j + t k$. Find the curvature of the path at the point where the particle is located at time $t = 0$.

27. Let $f(x, y) = x^2e^y$. Find the maximum value of a directional derivative at $(-2, 0)$ and find the unit vector in the direction in which the maximum value occurs.

28. Let $\vec{u} = 2i - j + 3k$, $\vec{v} = j + 7k$, $\vec{w} = i + 4j + 5k$. Find $(\vec{u} \times \vec{v}) \times (\vec{v} \times \vec{w})$.

29. Determine whether $\vec{F}(x, y) = e^x \sin y\, i + e^x \cos y\, j$ is a conservative field. If so, find a potential function for it.

30. Evaluate $\oint_C (e^x + y^2)\, dx + (e^y + x^2)\, dy$ using Green's theorem, where C is the boundary of the region between $y = x^2$ and $y = x$.

31. Use Stoke's theorem to evaluate $\oint_C \vec{F} . d\vec{r}$ where $\vec{F}(x, y, z) = z^2 i + 2xj - y^3k$ and C is the circle $x^2 + y^2 = 1$ in the xy – plane with counterclockwise orientation looking down the positive z-axis.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32\. a) Find the volume of the solid that is bounded above and below by the sphere $x^2 + y^2 + z^2 = 9$ and inside the cylinder $x^2 + y^2 = 4$.

b) Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 6 - x$ and $y = 0$ is revolved about the x-axis.

33\. a) A particle moves through 3 – space in such a way that its velocity is

$\vec{v}(t) = i + tj + t^2k$. Find the coordinates of the particle at time $t = 1$ given that the particle is at the point $(-1, 2, 4)$ at time $t = 0$.

b) Prove that $\text{div}(\phi \vec{F}) = \phi \text{div} \vec{F} + \nabla \phi . \vec{F}$.

34\. a) Find the work done by the force field $\vec{F}(x, y) = (e^x - y^3)i + (\cos y + x^3)j$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counterclockwise direction.

b) Use the Divergence theorem to find the outward flux of the vector field $\vec{F}(x, y, z) = x^3i + y^3j + z^2k$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.

35\. Find the flux of the vector field $\vec{F}(x, y, z) = z\,k$ across the outward oriented sphere $x^2 + y^2 + z^2 = a^2$.