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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme Under CBCSS

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – CALCULUS WITH APPLICATIONS IN PHYSICS – II

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all ten** questions are compulsory, each carries 1 mark each :

1. Find the modulus of $z = -4 + 3i$.

2. Define principal argument of a complex number.

3. Find all second order partial derivatives of $f(x, y, z) = (x + y)(3x + y)$.

4. Find total derivatives of $z = xe^{x-y}$.

5. Verify the exactness of the differential $xdy + 3ydx$.

6. Show that $(y + z)dx + xdy + xdy + xdz$ is an exact differential form.

7. Find $\int_0^1 \int_0^2 \int_0^3 dx\, dy\, dz$.

P.T.O.

8. If $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + 3t^3\hat{k}$ then $\frac{d\vec{r}}{dt}$.

9. If $F(x,y,z) = xyz\,\hat{i} + yz\hat{j} + xz\hat{k}$. Find $DivF$ at $(-1, 1, 1)$.

10. If $F(x,y,z) = (x+2)\hat{i} + y\hat{j} + z\hat{k}$. Find $curl\,F$. **(10 × 1 = 10 Marks)**

SECTION – II

Answer any **eight** questions, each carries 2 marks each :

11. Verify the result $|z_1 + z_2| = |z_1| + |z_2|$ for $z_1 = -2 + 4i$ and $z_2 = 2 + 3i$.

12. Find conjugate z^* of $z = w^{3y+2ix}$ where $w = x + yi$.

13. If $z = re^{i\theta}$, then find the value of $z^n + \frac{1}{z^n}$.

14. Show that $\cos ix = \cosh x$.

15. Show that $\cosh^2 x - \sinh^2 x = 1$.

16. Given that $x(u) = 1 + au$ and $y(u) = bu^3$, where a & b are constants. Find rate of change of $f(x,y) = xe^{-y}$ with respect u.

17. Show that the function $f(x,y) = xy$ has a saddle point at $(0, 0)$.

18. Suppose $f(x,y,z)$ be a function defined on a region V in space. Write the formula to find the average value of f in V and hence calculate average of $f(x,y,z) = 8xyz$ over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$.

19. Find $\int_0^1\int_0^x xy\,dy\,dx$.

20. If $\varphi(x, y, z) = xyz$, find $grad\, \varphi$.

21. Find Laplacian of $\varphi(x, y) = e^x \cos y$.

22. Verify whether the vector field $F(x, y, z) = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is irrotational or not.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions, each carries 4 marks each :

23. Find the principal values of (a) $z = \ln(-i)$ (b) $z = i^{-2i}$.

24. Find the solution of the equation $z^3 = 1$.

25. Solve the hyperbolic equation $\cosh x - 5\sinh x - 5 = 0$.

26. Find the Taylor's theorem to find a quadratic approximation of $f(x, y) = xe^y$ about the origin.

27. The temperature of a point (x, y) on a unit circle is given by $T(x, y) = 1 + xy$ find the temperature of the two hottest point on the circle.

28. Find the area enclosed by the curves $y = x^2$ and $x = y^2$ using the concept of double integral.

29. If $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$ and $z = r\cos\theta$ then find $\dfrac{\partial(x, y, z)}{\partial(r, \theta, \varphi)}$.

30. Show that $curl(grad\varphi) = 0$ for any scalar field φ.

31. If $F(x, y, z) = x^2y^2z^2\hat{i} + y^2z^2\hat{j} + x^2z^2\hat{k}$ then find $\nabla \cdot (\nabla \times F)$. **(6 × 4 = 24 Marks)**

SECTION – IV

Answer any **two** questions, each carries 15 marks each :

32. Find a closed-form expression for the inverse hyperbolic functions.

    (a) $y = \sinh^{-1} x$

    (b) $y = \tan^{-1} x$

    (c) $\frac{d}{dx}\sinh^{-1} x$.

33. (a) Find the point $P(x, y, z)$ closest to origin lies on the plane $2x + y - z = 5$.

    (b) The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the point on the ellipse that lies closest and farthest from origin.

34. (a) Evaluate the double integral $I = \iint_R (a + \sqrt{x^2 + y^2})\,dx\,dy$ where R is the region bounded by the circle $x^2 + y^2 = a^2$.

    (b) Find the average value of the function $f(x, y, z) = xyz$ over the cube bounded by the coordinate planes, $x = 2$, $y = 2$ and $z = 2$.

35. (a) If φ is any scalar field and A is any vector field, show that $\nabla \times (\varphi A) = \nabla\varphi \times A + \varphi\nabla \times A$.

    (b) Show that $\nabla \cdot (\nabla\phi \times \nabla\varphi) = 0$ where ϕ and φ are scalar fields.

    (c) If $r = \left[x\hat{i} + y\hat{j} + z\hat{k}\right]$, the find ∇r^n, for $n \in N$. **(2 × 15 = 30 Marks)**