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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS – II — CALCULUS WITH APPLICATIONS IN PHYSICS — II

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

PART – A

All **ten** questions are compulsory. Each question carries **1** mark.

1. Find the complex conjugate of $a + 2i + 3i\,b$.
2. State de Moivre's theorem.
3. Find the total differential of the function $f(x, y) = x\exp(x + y)$.
4. Check whether $xdy + 2ydx$ is exact or not?
5. Write down the necessary condition for a stationary point of the function $f(x, y)$.
6. Write down the formula for Jacobian.
7. Evaluate $\int_0^2\int_0^1\int_0^3 dz\,dx\,dy$.
8. Find derivative of $r(t) = t^2\, i + e^t\, j - (2\cos\pi t)\, k$.

P.T.O.

9. Find gradient of $f(x, y) = (x + y)$.

10. Let ϕ be a scalar function. Then *curl* (*grad* ϕ) is ————.

PART – B

Answer **any eight** questions from 11 to 26. Each question carries **2** marks.

11. Express $\sin(3\theta)$ and $\cos(3\theta)$ in terms of powers of $\cos\theta$ and $\sin\theta$.

12. Evaluate $Ln(-i)$.

13. Express $z = \frac{1}{1+i}$ in terms of $x + iy$.

14. Find $f_x(1,3)$ for the function $f(x,y) = 2x^3y^2 + 2y + 4x$.

15. Write down Taylor's theorem expansion of a function $f(x, y)$.

16. Find the stationary points of $f(x, y) = 3xy - 6x - 3y + 7$.

17. Show that $f(x, y) = x^2 + y^2$ has a minima at $(0, 0)$.

18. Show that $f(x, y) = -x^2 - y^2 + 25$ has a maxima at $(0, 0)$.

19. Evaluate $\int_1^3 \int_2^4 40 - 2xy \, dy \, dx$.

20. Find Jacobian of $x = \rho\cos(\phi)$, $y = \rho\sin(\phi)$ with respect to ρ and ϕ.

21. Evaluate the triple integral $\int_{-1}^2 \int_0^3 \int_0^2 12xy^2z^3 \, dz \, dy \, dx$.

22. Write down the formula for the centre of mass of a solid or laminar body.

23. Find the divergence of the vector field $a = x^2y^2i + y^2z^2j + x^2z^2k$.

24. Find curl of the vector field $F = x^2yi - (z^3 - 3x)j + 4y^2k$.

25. Show that $\text{curl}(r) = 0$, where $r = xi + yj + zk$.

26. The position vector of a particle at time t is given by $r(t) = 2\cos t\, i + 2\sin t\, j$. Find velocity of the particle.

PART – C

Answer **any six** questions from 27 to 38. Each question carries **4** marks.

27. Solve hyperbolic equation $\cosh x - 5\sinh x - 5 = 0$.

28. Find fourth root of i.

29. Show that $(y+z)\,dx + x\,dy + x\,dz$ is exact.

30. Consider the sphere $x^2+y^2+z^2=1$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.

31. Locate local maxima and minima of the function $f(x, y) = x^3 \exp\left(-x^2 - y^2\right)$.

32. Find the Taylor expansion, up to quadratic terms $x-2, y-3$ of $f(x, y) = y\exp(xy)$ about the point $x=2, y=3$.

33. Evaluate $\iint_R \left(2x - y^2\right) dA$ over the triangular region R enclosed between the lines $y=-x+1$, $y=x+1$ and $y=3$.

34. Find the volume of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=2y$.

35. Compute the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ where $x=\frac{v}{u}, y=u^2-4v^2$.

36. Find the Laplacian of scalar field $\phi = xy^2z^3$.

37. Prove that $curl\,(grad\ \phi) = 0$.

38. Find $r_\phi \times r_\theta$ where $r = a\sin\phi\cos\theta\, i + a\sin\phi\sin\theta\, j + a\cos\phi\, k$.

PART – D

Answer **any two** questions out of questions 39 to 44. Each question carries **15** marks.

39. (a) Solve the equation $z^6 - z^5 + 4z^4 - 6z^3 + 2z^2 - 8z + 8 = 0$. **7**

    (b) Find value of $z = i^{-2i}$. **8**

40. (a) Compute the total differential of $f(x, y, z) = x\sin(yz)$. **7**

    (b) Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$. **8**

41. (a) Evaluate double integral $I=\iint_R (a+\sqrt{x^2+y^2}\, dx\, dy$, where R is the region bounded by the circle $x^2+y^2=a^2$. **7**

(b) Find the mass of tetrahedron bounded by the three coordinate surfaces and the plane $\frac{x}{2}+\frac{y}{2}+\frac{z}{2}=1$, if its density is given by $3\left(1+\frac{x}{2}\right)$. **8**

42. (a) The position vector of a particle at time t is given by $r(t)=2t^2 i+(3t-2)j+(3t^2-1)k$. Find the speed of the particle at $t=1$ and the component of its acceleration in the direction $s=i+2j+k$. **7**

(b) Show that the divergence of $F(x, y, z)=\frac{c}{(x^2+y^2+z^2)^{\frac{3}{2}}}(xi+yj+2k)$ is zero. **8**

43. (a) By integrating $e^{(1+i)x}$ and separating real and imaginary parts, find the integrals of $e^x \cos x$ and $e^x \sin x$. **7**

(b) Derive the conditions for maxima for a function of two real variables. **8**

44. (a) Evaluate the integral $I=\int_{-\infty}^{\infty} e^{-(x^2)}\, dx$. **7**

(b) A triangular lamina with vertices $(0, 0)$, $(0, 1)$ and $(1, 0)$ has density function $\rho(x, y)=xy$. Find its total mass. **8**