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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2019

First Degree Programme under CBCSS

Complementary Course for Physics

MM 1131.1 : MATHEMATICS I – DIFFERENTIATION AND ANALYTIC GEOMETRY

(2014-2017 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – I

(All the first **ten** questions are compulsory. They carry 1 marks each)

1. Find the parametric equations of a circle with radius 2, centered at the origin, oriented counter clockwise.

2. Define a horizontal asymptote of a rational function.

3. Evaluate $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$.

4. State Rolle's theorem.

5. Find the Maclaurin's series for $\cos x$.

6. Use Euler's theorem to prove $x\,\partial u/\partial x + y\,\partial u/\partial y = 2u$ when $u = x^2 + xy + y^2$.

7. What is the eccentricity of a parabola?

8. Find the rectangular coordinates of the point whose polar coordinates are $(r, \theta) = \left(1, \frac{\pi}{4}\right)$.

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9. What are the asymptotes of the hyperbola $16x^2 - 25y^2 = 400$.

10. State the reflection property of hyperbolas.

SECTION – II

(Answer **any eight** from among the questions 11 to 22. These questions carry **2** marks each).

11. Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ of which one root is $-1+i$.

12. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude but opposite in sign.

13. Find the value of the constant A so that $y = A\sin 3t$ satisfies the equation $d^2y/dx^2 + 2y = 4\sin 2t$.

14. If the position function of a particle moving along a coordinate line is given by $s(t) = 12t - t^3$ where s is in feet. Find the velocity and acceleration after 3 seconds. Also find when and where are the velocity is a maximum.

15. Two particles A and B are in motion in the xy-plane. Their co-ordinates at each instant of time $t(t \geq 0)$ are given by $X_A = t$ and $X_B = 1-t$ and $Y_A = 2t$ and $Y_B = t$. Find the minimum distance between A and B.

16. Verify Mean Value Theorem for the function $f(x) = x^2 - 3x - 1$ in $[1, 3]$.

17. Find the radius of convergence and interval of convergence of the power series

$$1 + (x-3) + (x-3)^2 + ... + (x-3)^k + ...$$

18. If $z = (x+y)/(x-y)$ find $\partial z/\partial x$ and $\partial z/\partial y$.

19. Find the Jacobian $\partial(x,y)/\partial(u,v)$ if $x = u + 4v$ and $y = 3u - 5v$.

20. Given that $z = x/y, x = 2\cos, u, y = 3\sin v$. Find $\partial z/\partial u$ and $\partial z/\partial v$ using chain rule.

21. Find the equation of the ellipse whose ends of the major axis are $(\pm 3,0)$ and ends of the minor axis are $(0,\pm 2)$.

22. What is the discriminant of the quadratic equation $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$. Use this to identify the graph of the equation $2x^2+y^2-4x-4=0$.

SECTION – III

(Answer **any six** from among the questions 23 to 31. These questions carry **4** marks each).

23. A robot moves in the positive direction along a straight line so that after t minutes its distance is $s=6t^4$ feet from the origin.

    (a) Find the average velocity of the robot over the interval $[2, 4]$

    (b) Find the instantaneous velocity at $t=2$.

24. Show that $\sinh^{-1}x=\ln\left(x+\sqrt{x^2+1}\right)$ and $d/dx\{\sinh^{-1}x\}=1/\sqrt{x^2+1}$.

25. An open box is to be made from a 16 inch by 30 inch piece of cardboard by cutting out equal squares of equal size from the four corners and bending up the sides. Find the maximum volume of the box.

26. Locate all relative extrema and saddle points of $f(x,y)=3x^2-2xy+y^2-8y$.

27. Find the absolute maximum and minimum values of $f(x,y)=3xy-6x-3y+7$ on the closed triangular region R with vertices (0, 0), (3, 0) and (0,5).

28. Find the Taylor's series expansion for $f(x)=lnx$ about $x_0=1$.

29. Sketch the graph of the ellipse $x^2+2y^2=4$, showing foci.

30. Describe the graph of the equation $y^2-8x-6y-23=0$.

31. The period T of a simple pendulum of length l is given by $T=2\pi\sqrt{l/g}$ where g is a constant. Find the maximum error in T due to possible errors up to 1% in l and 2.5% in g.

SECTION – IV

(Answer **any two** from among the questions 32 to 35. These questions carry **15** marks each).

32. (a) Let V be the volume of a cylinder having height h and radius r, assume that h and r are vary with time. How are $dV/dt, dh/dt$ and dr/dt related?

(b) At a certain instant the height is 6 in increasing at 1 in/s, while the radius is 10 in and decreasing at 1 in/s. How fast is the volume changing at that instant? Is the volume increasing or decreasing at that instant?

(c) Water is running out of a conical funnel at the rate of 1 cc/sec. The radius of the base funnel is 4 cm, and the altitude is 16 cm. Find the rate at which the water level is dropping when it is 2 cm from the top.

33. (a) If $u = x^3y^3/(x^3+y^3)$, show that $x\partial u/\partial x + y\partial u/\partial y = 3u$.

(b) If $u = x^2\tan^{-1}(y/x) - y^2\tan^{-1}(x/y)$, then show that $\partial^2 u/\partial x\,\partial y = (x^2-y^2)/(x^2+y^2)$.

(c) Verify Euler's theorem for the function $u = x^3+y^3+z^3+3xyz$.

34. (a) Derive the equation of a parabola in the form $y^2 = 4px$.

(b) Prove the equation of the tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ at (x_1, y_1) is $xx_1/a^2 + yy_1/b^2 = 1$.

(c) Find the equation of the hyperbola with vertices $(0, \pm, 8)$ and asymptotes $y = \pm 4x\sqrt{3}$.

35. (a) Derive the polar equations for the conic section in the form $r = ed/(1 - e\cos\theta)$ from their focus- directrix property.

(b) Sketch the graph of $r = 2/(1-\cos\theta)$.

(c) State Kepler's first, second and third laws of planetary motion.