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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2019

First Degree Programme under CBCSS

Complementary Course for Physics

MM 1131.1: MATHEMATICS I — CALCULUS WITH APPLICATIONS IN PHYSICS – I

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

PART – I

Answer **all** questions. Each question carries **1** mark.

1. Find the derivative with respect to x of $f(t) = 2at$, where $x = at^2$.

2. State Mean value theorem.

3. Define the radius of curvature of the function.

4. Evaluate $\int x \sin x dx$

5. Give the formula for finding the area of a sector defined by the polar curve $\rho = \rho(\phi)$ and the radii vectors $\phi = \phi_1, \phi = \phi_2$.

6. Find the mean value of the function $f(x) = x$ over the interval [0,1].

7. Find the vector of length 2 that makes an angle of $\frac{\pi}{4}$ with positive x-axis.

8. Find a unit vector normal to both the vectors $\vec{a} = 4\hat{i} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j}$.

9. Determine whether the planes $3x-4y+5z=0$ and $-6x+8y-10z-4=0$ are parallel.

10. Define the absolute and conditional convergence of an infinite series.

(10 × 1 = 10 Marks)

PART – II

Answer **any eight** questions. Each question carries **2** marks.

11. Find the third derivative of the function $f(x)=(x^3+x)\cos x$, using Leibnitz' theorem.

12. Verify Rolle's theorem for the function $f(x)=\sin x$ on $[0,2\pi]$.

13. Find the inflection points, If any, of $f(x)=x^4$.

14. Evaluate $\int x^3 e^{-x^2} dx$.

15. Find the length of the curve $y=x^{3/2}$ from $x=0$ to $x=2$.

16. Find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval [1,4] is revolved about the x-axis.

17. Find the angle between a diagonal of a cube and one of its edges.

18. Find the parametric equation of the line through (2,–1,5) which is parallel to the vector $-\hat{i}+2\hat{j}+7\hat{k}$.

19. Find the distance between the lines $L_1 : x=2t,\ y=3+4t,\ z=2-6t$ and

    $L_2 : x=1+t,\ y=6t,\ z=-9t$.

20. Find an equation of the plane passing through the point (3,–1,7) and perpendicular to the vector $\vec{n}=4\hat{i}+2\hat{j}-5\hat{k}$.

21. Find the intersection of the line $x = 3 + 8t, y = 4 + 5t, z = -3 - t$ and the plane $x - 3y + 5z = 12$.

22. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)}$.

(8 × 2 = 16 Marks)

PART – III

Answer **any six** questions. Each question carries **4** marks

23. Find the positions and natures of the stationary points of the function $f(x) = \sin ax$ with $a \neq 0$.

24. Find the curvature of a smooth parametric curve $x = t$, $y = \frac{1}{t}$ at the point $t = 1$.

25. Evaluate $\int_0^1 \frac{x^3 + 1}{x^4 + 4x + 1} dx$.

26. Find the area of the region in the first quadrant that is within the cardioid $\rho = 1 - \cos\phi$.

27. Find the position vector of the centroid of the triangle with vertices A, B, C having position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively with respect to origin O.

28. Find the area of a triangle which is determined by the points $P_1(2,2,0)$, $P_2(-1,0,2)$ and $P_3(0,4,3)$.

29. Find the parametric equation of the line that contains the point $P(0,2,1)$ and intersect the line $L: x = 2t, y = 1 - t, z = 2 + t$ at right angle.

30. Consider a ball that drops from a height of $27\,m$ and on each bounce retains only a third of its kinetic energy; thus after one bounce it will return to a height of $9m$, after two bounces to $3\,m$ and so on. Find the total distance travelled between the first bounce and M^{th} bounce.

31. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(n!)^2} = 1 + \frac{1}{2^2} + \frac{1}{6^2} + \ldots$

(6 × 4 = 24 Marks)

PART – IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) If $I_n = \int_0^\infty x^n e^{-x} dx$ and if n is any positive integer, show that $I_n = nI_{n-1}$. Hence prove that $\int_0^\infty x^n e^{-x} dx = n!$. **8**

(b) Find the surface area of a cone formed by rotating about the x–axis the line $y = 2x$ between $x = 0$ and $x = h$. **7**

33. (a) Find the distance from the point $P(1, 4, -3)$ to the line $L: x = 2 + t, y = -1 - t, z = 3t$. **5**

(b) A line is inclined at equal angles to the x, y and $z-$axes and passes through the origin. Another line passes through the point (1, 2,4) and (0,0,1). Find the minimum distance between the two lines. **10**

34. (a) Find an equation of the plane that contains the line $x = 3t, y = 1 + t, z = 2t$ and is parallel to the intersection of the planes $2x - y + z = 0$ and $y + z + 1 = 0$. **7**

(b) Sum the series $S(x) = \frac{x^4}{3(0!)} + \frac{x^5}{4(1!)} + \frac{x^6}{5(2!)} + \ldots$ **8**

35. (a) If you invest Rs. 1000 on the first day of each year, and interest is paid at 5% on your balance at the end of each year, how much money do you have after 25 years. **5**

(b) Find the Maclaurin series for

(i) $\ln\left(\frac{1+x}{1-x}\right)$;

(ii) $\sin^2 x$. **10**

(2 × 15 = 30 Marks)