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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme under CBCSS
MATHEMATICS
MM 1661.3 : Complex Integration (Elective)
(2013 Admn.)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carries **1** mark **each.**

1. Evaluate $\int_0^\infty e^{-zt}dt$ (Rez > 0).

2. Evaluate $\int_C \frac{e^z}{z+2i}dz$ where C is the circle $|z-i| = 2$.

3. Let C be the circle $|z| = 3$ described in the positive sense. Show that if $g(z) = \int_C \frac{2s^2 - s - 2}{s - z} ds$, then $g(2) = 8\pi i$.

4. Find the singular point of the function $\frac{1}{z}$ and specify the type of singularity.

5. What type of singularity that $f(z) = \exp\left(\frac{1}{z}\right)$ have at the origin ?

6. The Laurent's series expansion of $f(z) = \exp\left(\frac{1}{z^2}\right)$ is $1 + \frac{1}{z^2} + \frac{1}{2!z^4} + \frac{1}{3!z^6} + \ldots$ about the point $z = 0$. Find the residue of f(z) at the isolated singular point $z = 0$.

7. Why $z = 0$ is not an isolated singular point of Log z ?

8. Find the residue at the simple pole $z = n\pi$ of the function $f(z) = \cot z$.

9. Find the singularities of the function $f(z) = \frac{\sin z}{z}$ and classify the singularity.

10. Write the Cauchy principle value (PV) of $\int_{-\infty}^{\infty} f(x)\,dx$, if f(x) is even.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each** :

11. Use an antiderivative F(z) of the integrand to evaluate $\int_0^{1+i} z^2\, dz$.

12. Find the length of the unit circle given by z(t) = cos t + isin t $(0 \leq t \leq 2\pi)$.

13. Evaluate $\int_C \bar{z}\, dz$ where C is the upper half of the unit circle |z| = 1 from z = – 1 to z = 1.

14. Let C denote the boundary of the square whose sides lie along the lines x = ±2 and y = ±2 where C is described in the positive sense.

    Evaluate $\int_C \frac{zdz}{2z+1}$.

15. Evaluate $\int_C \frac{e^{iz}}{z^3} dz$ where C is the circle |z| = 1.

16. If a function f is continuous throughout a simply connected domain, and if, for each simple closed contour C lying in D, $\int_C f(z)dz = 0$, then f is analytic throughout D.

17. Determine and classify the singular points of $f(z) = \frac{z}{e^z - 1}$.

18. Using Cauchy's integral formula, evaluate $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3} dz$ where C is |z| = 4.

19. Write the principal part of the function $f(z) = \sin\left(\frac{1}{z}\right)$ at its isolated singular point and determine the type of singularity.

20. Find the residue of $f(z) = \frac{z+1}{z^2 - z}$ at its pole z = 0.

21. Use Residue Theorem to evaluate $\int_C \frac{dz}{z^2 e^z}$ where C is the circle $|z| = 1$.

22. Find the residue at $z = 0$ of the function $f(z) = z \cos \frac{1}{z}$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Without evaluating the integral $\int_C \frac{dz}{z^4}$, determine the upper bound for its absolute value where C is the line segment from $z = 1$ to $z = i$.

24. State and prove Liouville's Theorem.

25. State and prove Cauchy's inequality.

26. Evaluate $\int_C \frac{z+2}{z} dz$ where C is the semicircle $z = 2e^{i\theta}$ where $0 \le \theta \le \pi$.

27. Prove that $\int_B \frac{dz}{z^2(z^2+9)} = 0$ where B consists of the circle $|z| = 2$ described in the positive direction together with the circle $|z| = 1$ described in the negative direction.

28. Find the residue of $f(z) = \frac{1}{(z^2+a^2)^2}$ at $z = ai$.

29. Find the residue at $z = 0$ of $\frac{1+e^z}{z \cos z + \sin z}$.

30. Find the residue of $f(z) = \frac{1}{z(e^z - 1)}$ at the origin.

31. Use residues to evaluate the definite integral $\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta}$.