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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2020

First Degree Programme under CBCSS

Mathematics

MM 1661.1 : GRAPH THEORY (Elective)

(2014 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Draw a graph with exactly 4 odd degree vertices.
2. How many 3-regular simple graphs are there with four vertices?
3. Define vertex disjoint subgraphs.
4. Define a walk in a graph with an example.
5. Define components of a graph.
6. Give an example of an Euler graph.
7. Does there exist a tree with minimum degree of its vertices is 2? Justify.
8. Draw a planar graph with 6 regions.
9. Write the adjacency matrix of the complete graph on 5 vertices.
10. Define a Balanced Digraph.

SECTION – II

Answer **any eight** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Prove that the sum of the degrees of all vertices in G is twice the number of edges of G.

12. State Konigsberg Bridge problem.

13. Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

14. Prove that a complete asymmetric digraph of n vertices contains $\frac{n(n-1)}{2}$ edges.

15. Sketch all non isomorphic simple digraphs with 3 vertices.

16. Define an Euler digraph and give an example.

17. Define weakly connected and strongly connected digraphs using examples.

18. Check whether the complete graph on 4 vertices is planar.

19. Give an example of a 3-connected planar graph.

20. Explain incidence matrix using an example.

21. Prove that there is one and only one path between every pair of vertices in a tree.

22. Draw the spanning trees of K_5.

SECTION – III

Answer **any six** questions from among the questions 23 to 31. These questions carry **4** marks each.

23. Prove that the number of vertices of odd degree in a graph is always even.

24. Prove that a graph G is disconnected iff its vertex set V can be partitioned into two non-empty disjoint sets V_1 and V_2 such that there exists no edge in G whose one end vertex in subset V_1 and the other in subset V_2.

25. Prove that if G is a connected graph with exactly $2k$ odd vertices, then there exist k edge-disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

26. Prove that a graph G with n vertices, $n-1$ edges and no circuits is connected.

27. Prove that every connected graph has at least one spanning tree.

28. Prove that for any simple, connected planar graph with f regions, n vertices and e edges, the following inequalities hold

(a) $e \geq \frac{3}{2} f$

(b) $e \leq 3n - 6$.

29. Prove that the Petersen graph is non-planar.

30. Explain the Teleprinter's problem and model the problem using graph theory.

31. Prove that two graphs G_1 and G_2 are isomorphic iff their incidence matrices $A(G_1)$ and $A(G_2)$ differ only by permutations of rows and columns.

SECTION – IV

Answer **any two** questions from among the questions 32 to 35.These questions carry **15** marks each :

32. Prove that a graph is a tree iff it is minimally connected.

33. Prove that a connected planar graph with n vertices and e edges has $e-n+2$ regions.

34. Prove that a digraph G is an Euler digraph iff G is connected and balanced.

35. Prove that every tree has either one or two centers.