(Pages : 4)

G – 1466

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme under CBCSS
MATHEMATICS
Elective
MM 1661.1 : Graph Theory
(2014 Admission Onwards)

Time : 3 Hours   Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. They carry **1** mark **each**.

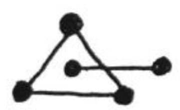
1. Define a simple graph.

2. The number of odd vertices in a graph is always __________

3. What is a spanning subgraph ?

4. Define outdegree.

5. Is the following graph connected ?



6. Define Euler graph.

7. What is a unicursal graph ?

8. Define radius of a graph.

9. A tree with n vertices has __________ edges.

10. What is maximal tree of a graph ?

P.T.O.

SECTION – II

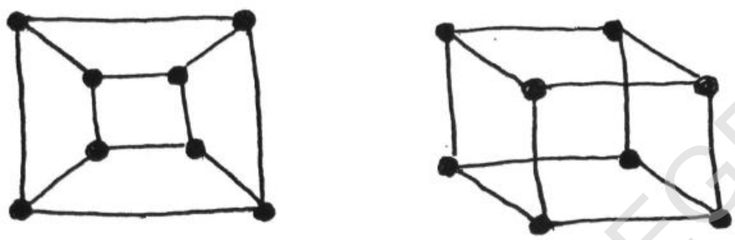
Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Write any two applications of graph theory with suitable explanation.

12. Write the adjacency matrix of C_4.

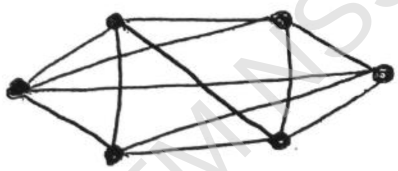
13. Prove that the sum of degrees is equal to twice the number of edges.

14. Label the following graphs to prove that they are isomorphic.



15. Prove that a graph G is disconnected if and only if the vertex set can be partitioned into 2 non-empty disjoint subsets V_1 and V_2 such that there is no edge having one end vertex in V_1 and another in V_2.

16. Is the following graph Euler graph ? Explain.



17. Explain Chinese Postman problem.

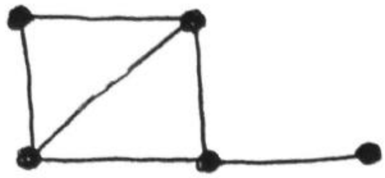
18. State a characterization theorem for Euler digraph. Illustrate with an example.

19. Prove that there is one and only one path between every pair of vertices in a tree T.

20. Prove that a graph with n vertices, n – 1 edges and no circuits is connected.

21. Prove that a graph G is a tree if and only if it is minimally connected.

22. Define spanning tree. Find a spanning tree of the following graph.



SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Draw all non-isomorphic graphs on 4 vertices. How many of them are self-complementary ? How many are connected ?

24. Define spanning subgraph and induced subgraph. Is P_4 a spanning subgraph of K_4 ? Is it an induced subgraph ? Explain.

25. Define incidence matrix. Draw the graph with incidence matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

26. Prove that a graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ components.

27. In a connected graph G with exactly 2k odd vertices, prove that there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

28. Prove that every tree has either one or two centers.

29. Prove that every connected graph has at least one spanning tree.

30. Draw planar representations of K_4 and a cube.

31. Prove that in any simple connected planar graph with f regions, n vertices and e edges, $e \geq \frac{3f}{2}$ and $e \leq 3n - 6$.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

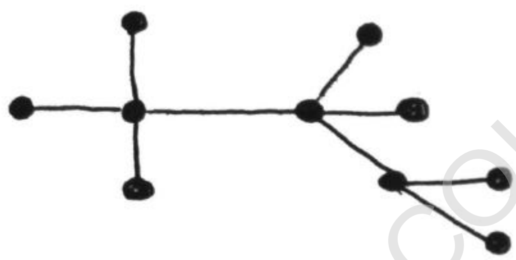
32. Explain in detail multicolour cube puzzle and its graph theoretic model.

33. Explain decanting problem with its graph theoretic formation.

34. a) Prove that, if in a graph G there is one and only one path between every pair of vertices, then G is a tree.

b) Prove that a tree with n vertices had n – 1 edges.

c) Find the center of the following tree.



35. Define planar graphs. State a necessary and sufficient condition for a graph G to be planar. Explain Four Colour Theorem and its graph theoretic interpretation.