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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

MM 1661.1 : Graph Theory (Elective)

(2018 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define a simple graph.
2. How many edges do K_{20} have?
3. Define an empty graph.
4. Define a connected graph.
5. A tree with n vertices has ... edges.
6. Define a cut vertex of a graph.
7. Define a Hamiltonian graph.
8. Is K_4 Eulerian?
9. How many regular polyhedra are there?
10. State Euler's formula in a connected plane graph.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Define a bipartite graph with example.

12. Define a regular graph. Draw a 2-regular graph.

13. Let G be a graph in which there is no pair of adjacent edges. What can you say about the degrees of the vertices in G?

14. Define complement of a graph. Give an example.

15. Define join of two graphs.

16. Define adjacency matrix of a graph G. Give an example.

17. Draw all non-isomorphic trees with 4 vertices.

18. Define vertex connectivity of a graph with example.

19. Define a maximal non-Hamiltonian graph. Give example.

20. Define an Euler tour. Give an example.

21. Define a Jordan curve.

22. State Cayley's theorem on spanning trees.

23. Define a plane graph. Give an example.

24. Define a polyhedral graph.

25. State Kuratowski's theorem on planar graphs.

26. How can we obtain a subdivision of a graph G.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4** marks each.

27. Prove that in any graph there is an even number of odd vertices.

28. Given two vertices u and v of a graph G. Prove that every u-v walk contains a u-v path.

29. Define the following in a connected graph

    (a) Distance between two vertices

    (b) Eccentricity of a vertex

    (c) Radius of a graph

    (d) Diameter of a graph.

30. Let u and v be distinct vertices of a tree T. Prove that there is precisely one path from u to v.

31. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G.

32. Let G be a graph with n vertices where $n \geq 2$. Prove that G has at least two vertices which are not cut vertices.

33. Describe Konigsberg bridge problem.

34. Define closure of a graph. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.

35. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.

36. Prove that the complete graph K_5 is non planar.

37. If G is a simple planar graph then prove that G has a vertex v of degree less than 6.

38. Let G be a simple graph with at least 11 vertices. Prove that either G or its complement $\overline{G}$ must be non-planar.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry **15** marks each.

39. Let G be a non-empty graph with at least two vertices. Prove that G is bipartite if and only if it has no odd cycles.

40. Define a spanning tree with example. Prove that a graph G is connected if and only if it has a spanning tree.

41. Let G be a simple graph with at least three vertices. Then prove that G is 2-connected if and only if for each pair of distinct vertices u and v of G there are two internally disjoint u-v paths in G.

42. Prove that a connected graph is Euler if and only if the degree of every vertex is even.

43. (a) Describe travelling salesman problem.

    (b) If G is a simple graph with n vertices where $n \geq 3$ and the degree $d(v) \geq n/2$ for every vertex V of G then prove that G is Hamiltonian.

44. Let P be a convex polyhedron and G be its corresponding polyhedral graph. For each $n \geq 3$ let v_n denote the number of vertices of G of degree n and let f_n denote the number of faces of G of degree n. Prove that.

    (a) $\sum_{n\geq 3} n v_n = \sum_{n \geq 3} n f_n = 2e$, where e is the number of edges of G.

    (b) The polyhedron P and so the graph G has at least one face bounded by a cycle of length n for either $n = 3, 4$ or 5.

(2 × 15 = 30 Marks)