(Pages : 4) **L – 1566**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2021

First Degree Programme Under CBCSS

Mathematics

Core Course XIII

MM 1645 — INTEGRAL TRANSFORM

(2018 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. These questions carry 1 mark **each**.

1. What are inverse transforms?
2. What is $L(t^n)$?
3. Write shifted function.
4. What is convolution of f and g, $f * g$?
5. If $L(f) = \dfrac{1}{(s^2 + \beta^2)^2}$ then $f(t) =$ ————
6. What is the period of sine function?
7. Give the Euler formula for a_n, where a_n is the coefficient of $\cos nx$ in the Fourier series expansion of a periodic function.
8. $\int_{-\pi}^{\pi} \sin nx \ \sin mx \ dx =$
9. Define odd function.
10. Write the Fourier series expansion of an even function.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Find Laplace transform of e^{at}.
12. Find the Laplace transform of cosh at.
13. Under suitable conditions, prove that $L(f'') = s^2 L(f) - sf(0) - f'(0)$.
14. Find the inverse transform of $\dfrac{1}{s(s^2 + \omega^2)}$.
15. Is the convolution, $f * 1 = f$? Justify.
16. Show that $f * g = g * f$.
17. What is Dirac delta function?
18. State existence theorem for Laplace transforms.
19. Prove that $\int_{-\pi}^{\pi} \cos nx \cos mx \; dx = 0$.
20. Are there functions which are neither even nor odd? Justify.
21. What is the amplitude spectrum of rectangular wave function?
22. Give the representation of a periodic function $f(x)$ as a Fourier integral.
23. What is the relation between Dirichlet' s discontinuous function and sine integral?
24. What is Gibbs phenomenon?
25. Represent $f(x) = 1/(1 + x^2)$ as an integral.
26. Write the Fourier cosine transform of an even periodic functions $f(x)$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4** marks **each**.

27. State and prove linearity of the Laplace transforms.
28. Find the inverse transform $L(f) = \dfrac{3s - 137}{s^2 + 2s + 401}$.

29. If $f(t) = t.\sin\omega t$, find $L(f)$.

30. Prove that $L\left(\int_0^t f(\tau)d\tau\right) = \frac{1}{s}F(S)$.

31. If $H(S) = \frac{1}{\left(s^2 + \omega^2\right)^2}$, find $h(t)$.

32. Solve Volterra's integral equation of second kind :

$$y(t) - \int_0^t (1+\tau)y(t-\tau)d\tau = 1 - \sin h\,\tau$$

33. Sketch the graph of $f(x) = |\sin x|$.

34. Write the Fourier series and Euler formula for the coefficients for a function $f(x)$ of period $2L$.

35. Find the Fourier sine series of $f(x) = \begin{cases} -k & if\ -2 < x < 0 \\ k & if\ 0 < x < 2 \end{cases}$ $P = 2L = 4,\ L = 2$.

36. Derive Fourier sine integral for $f(x) = e^{-kx}$ for $x > 0$, $k > 0$.

37. Find the Fourier cosine transforms of the function $f(x) = \begin{cases} k & if\ 0 < x < a \\ 0 & if\ x > a \end{cases}$.

38. Find $\mathcal{F}_c\left(e^{-x}\right)$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry **15** marks each.

39. (a) Solve the initial value problem $y'' + y' + 9y = 0$, $y(0) = 0.16$, $y'(0) = 0$.

(b) Determine the response of the damped mass-spring system under a square wave modeled by $y'' + 3y' + 2y = r(t) = u(t-1) - u(t-2)$, $y(0) = 0$, $y'(0) = 0$.

40. (a) Solve $y'' + y' = 2t$, $y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$, $y'\left(\frac{\pi}{4}\right) = 2 - \sqrt{2}$.

(b) State and prove Convolution Theorem.

41. Solve the initial value problem for a damped mass-spring system acted upon by a sinusoidal force for some time interval $y'' + 2y' + 2y = r(t)$, $r(t) = 10.\sin 2t$, if $0 < t < \pi$ and if $t > \pi$; $y(0) = 1$, $y'(0) = -5$.

42. Find the Fourier series expansion of $f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$.

43. (a) Find the Fourier series expansion of sawtooth wave function.

(b) Let $f(x)$ be continuous and absolutely integrable on the x-axis. $f'(x)$ is piecewise continuous on every finite interval and let $f(x) \to 0$ as $x \to \infty$. Prove that

(i) $\mathcal{F}_c|f'(x)| = w\,\mathcal{F}_s[f(x)] - \sqrt{\frac{2}{\pi}}\,f(0)$

(ii) $\mathcal{F}_s|f'(x)| = -\omega\,\mathcal{F}_c|f(x)|$.

44. Find the two half range expansions of $f(x) = \begin{cases} \frac{2k}{L}x, & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) & \text{if } \frac{L}{2} < x < L \end{cases}$.

(2 × 15 = 30 Marks)
