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E – 1630

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2018
First Degree Programme under CBCSS
MATHEMATICS
Core Course X
MM 1642 : Linear Algebra
(2013 Admn.)

Time : 3 Hours   Max. Marks : 80

SECTION – I

All questions are **compulsory**. They carry **1** mark **each**.

1. Show by an example that the matrix multiplication is not commutative.
2. Define a nilpotent matrix.
3. Give an example of a matrix whose inverse is the same as its transpose.
4. Define a Skew-Hermitian matrix.
5. Define a Vector Space.
6. Define the rank of a linear mapping.
7. Define the determinant of a matrix.
8. Define a permutation.
9. Define the eigen vector corresponding to the eigen value of a matrix.
10. Define the signature of a permutation.

SECTION – II

Answer **any 8** questions. They carry **2** marks **each**.

11. Prove that the matrix multiplication is associative.
12. Prove that if A, B are commutative, then A^T, B^T are commutative.

P.T.O.

13. Find the equation of the curve obtained by rotating the hyperbola, $x^2 - y^2 = 1$, through 45° anti-clockwise about the origin.

14. State the conditions for a system of linear equations, AX = B, to be consistent.

15. Define the left and the right inverses of a matrix.

16. Let V be a vector space, W be a subspace of V and $f : V \to W$ be a linear mapping. Then prove that $f(-x) = -f(x)$ for all $x \in V$.

17. Prove that if A is invertible, then $\det A^{-1} = \dfrac{1}{\det A}$.

18. State a characterizations of Similar matrices in terms of linear mappings.

19. Define the linear span of a set of elements of a vector space.

20. Define a determinantal mapping.

21. Define the adjoint of a matrix.

22. When a linear mapping is diagonalizable ?

SECTION – III

Answer **any 6** questions. They carry **4** marks **each**.

23. Show that the rows of the matrix, $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ are linearly independent.

24. Define the linear span of a set of elements of a vector space, V and prove that it is a subspace of V.

25. Use matrix notation to solve the Fibonacci Seers : $a_{n+2} = a_{n+1} + a_n$, $a_0 = 0$, $a_1 = 1$.

26. Use Gaussian elimination to solve the following system of linear equations :

    $x + z = 1$; $x + y = 2$; $3x + y + z + w = 1$; $y + 2z + w = 2$.

27. Explain the matrix representation of complex numbers.

28. If V, W are vector spaces of dimensions m, n respectively and $f : V \to W$, $g : V \to W$ are linear mappings, then matrix of $f \circ g$ = matrix of f × matrix of g with respect to the fixed ordered bases, where $f \circ g$ is the composition of f and g.

29. Prove that a non-empty subset S of a vector space V is a basis of V if and only if any element of V can be uniquely represented as a linear combination of elements of S.

30. Prove that the eigen vectors corresponding to distinct eigen values of a matrix are linearly independent.

31. Prove that any permutation on a set with $p > 2$ elements is a composition of transpositions.

SECTION – IV

Answer **any 2** questions. They carry **15** marks **each**.

32. Find the values of k such that the following system of linear equations has a solution

$$x + y + z = 1$$
$$x - 2y + z = 1$$
$$x + 2y + z = k.$$

33. Show that a 2 × 2 matrix is orthogonal if and only if it is of the form,

$$\begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} \text{ or } \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix}, \text{ where } \alpha^2 + \beta^2 = 1$$

34. Show that the set V of complex matrices of the form, $\begin{bmatrix} x & y \\ z & -w \end{bmatrix}$ is a real vector space of dimension, 6.

35. Diagonalize the Matrix, $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$.