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E – 1703

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2018
First Degree Programme under CBCSS
MATHEMATICS
Core Course – IX
MM 1641 : Real Analysis – II
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example of a function $f : [0, 1] \to \mathbb{R}$ that is discontinuous at every point of $[0, 1]$ but such that $|f|$ is continuous on $[0, 1]$.
2. Let $f : A \to \mathbb{R}$ for $A \subset \mathbb{R}$. State the sequential criterion for continuity of f at $a \in A$.
3. If $f : \mathbb{R} \to \mathbb{R}$ is continuous and takes only rational values then f is constant. True or false ?
4. Evaluate $\lim_{x\to 0}\left[\frac{1-\cos x}{x^2}\right]$.
5. State the Interior Extremum Theorem.
6. Let $f : (a, b) \to \mathbb{R}$ and $c \in (a, b)$. Explain what is meant by the statement, "f is differentiable at x = c" ?
7. Suppose that f has an inverse and $f(2) = -4$, $f'(2) = 3/4$. If $g = 1/f^{-1}$, what is $g'(-4)$?
8. Find $F'(x)$ when F is defined on [0, 1] by $F(x) = \int_0^{x^2} (1+t^3)^{-1}dt$.
9. If $f(x) = x^2$ for $x \in [0, 4]$, calculate the Riemann sum, where P = {0, 1, 2, 4} and the tags are selected at the left end points of the sub-intervals.

P.T.O.

10. Let $f \in \mathcal{R}[a, b]$ and define $F(x) = \int_a^x f$ for $x \in [a, b]$. Evaluate $S(x) = \int_x^{\sin x} f$ in terms of F.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Let $A, B \subseteq \mathbb{R}$ and let $f : A \to \mathbb{R}$ and $g : B \to \mathbb{R}$ be functions such that $f(A) \subseteq B$. If f is continuous at a point $c \in A$ and g is continuous at $b = f(c) \in B$, show that the composition $g \circ f : A \to \mathbb{R}$ is continuous at c.

12. Let I be an interval and let $f : I \to \mathbb{R}$ be continuous on I. If $a, b \in I$ and if $k \in \mathbb{R}$ satisfies $f(a) < k < f(b)$, show that there exists a point $c \in I$ between a and b such that $f(c) = k$.

13. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that the set $f(I)$ is a closed bounded interval.

14. Suppose that f is continuous on a closed interval $I = [a, b]$, that the derivative f' exists at every point of the open interval (a, b), and that $f(a) = f(b) = 0$. Show that there exists at least one point c in (a, b) such that $f'(c) = 0$.

15. Let I be an interval and let $f : I \to \mathbb{R}$ be differentiable on I. Show that if f' is positive on I, then f is strictly increasing on I.

16. If f is differentiable on $I = [a, b]$ and if k is a number between $f'(a)$ and $f'(b)$, show that there is at least one point c in (a, b) such that $f'(c) = k$.

17. Let $n \in \mathbb{N}$ and let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^n$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$. For which values of n is f' continuous at 0 ? For which values of n is f' differentiable at 0 ?

18. Evaluate $\lim_{x \to \infty} (1 + 1/x)^x$.

19. Prove that $e^{\pi} > \pi^{e}$.

20. If $f \in \mathcal{R}[a, b]$ and $|f(x)| \leq M$ for all $x \in [a, b]$, show that $\left| \int_a^b f \right| \leq M(b - a)$.

21. If f is continuous on [a, b], a < b, show that there exists c ∈ [a, b] such that we have $\int_a^b f = f(c)(b-a)$.

22. Applying the fundamental theorem, show that there does not exist a continuously differentiable function f on [0, 2] such that f(0) = – 1, f(2) = 4, and f′ (x) ≤ 2 for 0 ≤ x ≤ 2.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Show by example that a function that is differentiable at every point of $\mathbb{R}$ need not have a continuous derivative.

24. State and prove the Caratheodory's theorem.

25. Let f be defined on $\mathbb{R}$ by

$$f(x)=\begin{cases} x^2+ax+b & \text{if } x\geq 0 \\ \sin x & \text{if } x<0 \end{cases}.$$

Is it possible to find a, b such that f is differentiable on $\mathbb{R}$? If not, explain why; if yes, give the values of a, b.

26. Find the points of relative extrema of the function $f(x) = |x^2 - 1|$ for $-4 \leq x \leq 4$.

27. State and prove the Taylor's theorem.

28. Evaluate $\lim_{x\to 0} \frac{x^2-\sin^2 x}{x^4}$.

29. If $f \in \mathcal{R}[a, b]$, show that f is bounded on [a, b].

30. State and prove the general form of Integration by Parts for the Riemann integral.

31. Show that every continuous function is Riemann integrable.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be strictly monotone and continuous on I. Show that the function g inverse to f is strictly monotone and continuous on f(I).

33. State and prove the Newton's method to estimate a solution of an equation.

34. a) If $f : [a, b] \to \mathbb{R}$ is monotone on [a, b], show that $f \in \mathcal{R}[a, b]$.

    b) Let $K(x) = x^2 \cos(1/x^2)$ for $x \in (0, 1]$ and let $K(0) = 0$. Find $K'(x)$ and check whether $K' \in \mathcal{R}[0, 1]$.

35. a) Show that, in general, the indefinite integral need not be an antiderivative.

    b) State and prove the fundamental theorem of calculus (second form).