(Pages : 3) **F – 2469**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2018
First Degree Programme under CBCSS
MATHEMATICS
Core Course VIII
MM 1545 : Abstract Algebra – I
(2014 Admn. Onwards)

Time : 3 Hours Max. Marks : 80

PART – A

All the **first 10** questions are **compulsory**. They carry **1** mark **each.**

1. Check whether addition of matrices is a binary operation on the set of all matrices with real entries.
2. Let F be the set of all functions f mapping $\mathbb{R}$ into $\mathbb{R}$ that have derivatives of all orders. Check whether $\phi : (F, +) \to (F, +)$ defined by $\phi(f) = f'$ is an isomorphism.
3. Show that the binary structure $(M_2(\mathbb{R}), \cdot)$ of 2×2 real matrices with usual matrix multiplication is not isomorphic to $(\mathbb{R}, \cdot)$.
4. Check whether $\mathbb{Z}^+$ under multiplication is a group.
5. Does there exist a non-abelian group of order 7. Give reasons.
6. How many subgroups are there for $\mathbb{Z}_{30}$?
7. What is the maximum number of elements there for a cyclic group G with only one generator, where $G \neq \{e\}$?
8. Show by an example that every proper subgroup of a non-abelian group may be abelian.
9. Find the orbits of the permutation $\sigma : \mathbb{Z} \to \mathbb{Z}$ given by $\sigma(n) = n + 2$.
10. Find the index of $\langle \mu_1 \rangle$ in the group S_3.

P.T.O.

PART – B

Answer **any eight** questions from this Part. **Each** question carries **two** marks.

11. Prove that the binary structure $\langle M_2(\mathbb{R}), \cdot\rangle$ of 2×2 real matrices with the usual matrix multiplication is not isomorphic to $\langle \mathbb{R}, \cdot\rangle$ with the usual matrix multiplication.

12. Determine whether the set of all $n \times n$ matrices with determinant -1 is a subgroup of $GL(n, \mathbb{R})$.

13. True or False : Every group of order ≤ 4 is cyclic. Justify.

14. Prove that if $a \in G$, the subgroups generated by a and a^{-1} are the same.

15. Prove that every cyclic group is abelian.

16. Prove that the collection of all even permutations of {1, 2, 3, ..., n) form a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n.

17. Compute $\tau^2\sigma$ in S_6 where $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$ and $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$.

18. What is the order of the permutation ?
(1 2 4) (3 5 7 8).

19. Prove that every group of prime order is cyclic.

20. Does there exist a subgroup of a group of order 6 whose left cosets give a partition of the group into 12 cells ?

21. Let G be a group and let H be a subgroup of G. Let $a, b \in G$. Prove that the number of elements in aH is the same as the number of elements in bH.

22. Find a generator of $\mathbb{Z}_2 \times \mathbb{Z}_3$.

PART – C

Answer **any six** questions. **Each** question carries **4** marks.

23. Prove that a non-empty subset H of a group G is a subgroup of G iff $a \in H$, $b \in H \Rightarrow ab^{-1} \in H$.

24. Describe all the elements in the cyclic subgroup of $GL(2, \mathbb{R})$ generated by the 2×2 matrix.

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

25. Let G be a cyclic group with generator a, and let G' be a group isomorphic to G. If $\phi : G \to G'$ is an isomorphism, show that for every $x \in G$, $\phi(x)$ is completely determined by the value of $\phi(a)$.

26. Describe the sub-group diagram for the dihedral group D_4.

27. Show that a cycle of length n has order n.

28. Let H be a subgroup of G. Let the relation $\sim_L$ be defined on G by

$a \sim_L b$ iff $a^{-1}b \in H$

Show that $\sim_L$ is an equivalence relation.

29. State and prove Lagrange's theorem for groups.

30. Show that if G is a group of even order, then it has an element $a \neq e$ satisfying $a^2 = e$.

31. Show that every permutation σ of a finite set is a product of disjoint cycles.

PART – D

Answer **any two** questions from this Part. **Each** question carries **15** marks.

32. Describe the group U_n and $GL(n, \mathbb{R})$ with their properties.

33. Prove that a subgroup of a cyclic group is cyclic.

34. State and prove Cayley's theorem.

35. The group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and is isomorphic to $\mathbb{Z}_{mn}$ iff m and n are relatively prime.