Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021.

First Degree Programme under CBCSS

Mathematics

Core Course VIII

MM 1544 – DIFFERENTIAL EQUATIONS

(2018 & 2019 Admission)

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Marks : 80

SECTION – I

Answer all questions :

1. Write the standard equation of linear differential equation.

2. Write the Lipschitz condition.

3. Solve $dy + y\,dx = 0$.

4. For what values of the constant m will $y = e^{mx}$ be the solution of $y'' - 3y' - 10y = 0$.

5. Check whether $y^2 dy + x^2 dx$ exact or not.

6. Find the complementary function of $\dfrac{d^2 y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^x \sin x$.

7. Define Wronskian

8. Write the standard form of Legendre's linear equation

9. Write the characteristic equation of $2\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 3y = 0$.

10. Define basis of solutions of a homogeneous second order differential equation

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions

11. Find the order and degree of the ODE $\dfrac{d^3y}{dx^3} + 2\left(\dfrac{dy}{dx}\right)^{\frac{1}{2}} = 0$

12. Define partial differential equation. Give one example of it

13. Solve $\dfrac{dy}{dx} = xy + x$.

14. State the uniqueness theorem of first order differential equation.

15. Verify that $y = \dfrac{2}{x}$ is a solution of the differential equation $xy' = -y$, for all $x \neq 0$.

16. Show that a seperable equation is also exact.

17. Check the exactness of $y' = 1 + y^2$.

18. Find the integrating factor of $y\,dx - x\,dy = 0$.

19. Find the general solution of $\dfrac{d^2y}{dx^2} + 4y = 0$.

20. Find a differential equation whose solution is cos 3x.

21. Find the complementary function of $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 3e^x$.

22. Write the basis of solution of the equation $\dfrac{d^2y}{dx^2} + y = 0$

23. Write the standard form of Euler- Cauchy equation. Give one example of it.

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24. Solve $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2 = 0$

25. Find a general solution of $x^2 y' - 20y = 0$.

26. Find the Wronskian of e^x and e^{-x}.

<div align="right">**(8 × 2 = 16 Marks)**</div>

SECTION – III

Answer **any six** questions

27. Solve $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

28. Solve $(x+4)(y^2+1)dx + y(x^2+3x+2)dy = 0$

29. Find the Orthogonal Trajectories of the family $cx^2 + y^2 = 1$

30. Solve the initial value problem $y' + y\tan x = \sin 2x, y(0) = 1$

31. Solve $x\dfrac{dy}{dx} + y = xy^{\frac{3}{2}}, y(1) = 4$.

32. Solve $(x^2 - 3y^2)dx + 2xy\,dy = 0$

33. By reducing the order, solve $(x^2+1)y'' - 2xy' + 2y = 0$, given x is one solution

34. Solve $\dfrac{d^2y}{dx^2} + y = \sin x$.

35. Find the general solution of the equation $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = 6e^x$.

36. Solve $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - 16y = 0$.

37. Solve the logistic equation $y' = Ay - By^2$.

38. Solve $y'' + y = \cosec\,x$ using the method of variation of parameters.

<div align="right">**(6 × 4 = 24 Marks)**</div>

<div align="right">**M – 1459**</div>

SECTION – IV

Answer **any two** questions

39. (a) Solve $\left(\dfrac{3-y}{x^2}\right)dx + \left(\dfrac{y^2-2x}{xy^2}\right)dy = 0$, $y(-1) = 2$ by exactness.

   (b) Find an integrating factor and solve

   $(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0$

40. (a) Solve the initial value problem $(ye^x + 2e^x + y^2)dx + (e^x + 2xy)dy = 0$, $y(0) = 6$

   (b) Find a basis of solutions of the differential equation $(x^2 - x)y'' - xy' + y = 0$.

41. (a) Check the exactness and solve $(2xy^2 + y)dx + (2y^3 - x)dy = 0$.

   (b) Solve the initial value problem $(y + \sqrt{x^2 + y^2})dx - xdy = 0$, $y(1) = 0$.

42. (a) Solve $x^2y'' - 2xy + 2y = 0$, $y(1) = 1$, $y'(1) = 1$.

   (b) Solve $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}$.

43. (a) Solve $(D^2 + 2D + \dfrac{3}{4}I)y = 3e^x + \dfrac{9}{2}x$.

   (b) Solve $y''' - 3y'' + 2y' = 0$.

44. (a) Solve the initial value problem

   $y'' - 2y' - 3y = 2e^x - 10\sin x$, $y(0) = 2$, $y'(0) = 4$

   (b) Solve $(D^2 + 3D + 2I)y = 5x^2$.

.(2 × 15 = 30 Marks)