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F – 2385

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2018
First Degree Programme under CBCSS
MATHEMATICS
Core Course
MM1541 : Real Analysis – I
[2013 Admn.]

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each.**

1. If $a \in \mathbb{R}$, prove that a . 0 = 0.

2. Determine the set A of all real numbers x such that $2x + 3 \leq 6$.

3. Define absolute value of a real number.

4. Define a sequence of real numbers.

5. Prove that $\lim \frac{1}{n} = 0$.

6. Give an example of an unbounded sequence that has a convergent subsequence.

7. State Monotone subsequence theorem.

8. Define absolute convergence of a series.

9. Find the cluster points, if any, of the set A = {1, 2, 3, 4, 5}.

10. Define right hand limit of a function.

P.T.O.

SECTION – II

Answer **any 8** questions among the questions **11** to **22**. These questions carry **2** marks **each**.

11. State order properties of $\mathbb{R}$.

12. If $a \in \mathbb{R}$ satisfies $a \cdot a = a$, prove that either $a = 0$ or $a = 1$.

13. Prove the triangle inequality $|a + b| \leq |a| + |b|$; $a, b \in \mathbb{R}$.

14. Let A and B be two non empty subsets of $\mathbb{R}$ such that $a \leq b$ for all $a \in A$ and for all $b \in B$. Prove that $\sup A \leq \inf B$.

15. If $X = (x_n)$ and $Y = (y_n)$ are convergent sequences of real numbers and if $x_n \leq y_n$ for all $n \in \mathbb{N}$, then prove that $\lim (x_n) \leq \lim (y_n)$.

16. Give an example of two divergent sequences X and Y such that their sum X + Y converges.

17. Prove that $\left(\frac{1}{n}\right)$ is a Cauchy sequence.

18. Prove that every Cauchy sequence of real numbers is bounded.

19. Prove that $\lim \left(\frac{1}{n^2+1}\right) = 0$.

20. If the series $\sum x_n$ converges, prove that $\lim x_n = 0$.

21. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$ converges.

22. Show that $\lim_{x \to 0} \frac{1}{x^2} = \infty$.

SECTION – III

Answer **any 6** questions among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Prove that there does not exists a rational number r such that $r^2 = 2$.

24. If $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, then prove that $\inf S = 0$.

25. Prove that a sequence of real numbers can have at most one limit.

26. Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively. Prove that the sequences X + Y and X . Y converge to x + y and xy respectively.

27. State and prove Squeeze theorem for sequences.

28. Prove that every Cauchy sequence is convergent.

29. Check whether the following series are convergent or not.

    a) $\sum_{n=1}^{\infty} \frac{1}{n!}$

    b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$

30. Prove that a number $c \in \mathbb{R}$ is a cluster point of a subset A of $\mathbb{R}$ if and only if there exists a sequence (a_n) in A such that $\lim(a_n) = c$ and $a_n \neq c \ \forall \ n \in \mathbb{N}$.

31. Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ has a limit at $c \in \mathbb{R}$. Prove that f is bounded on some neighbourhood of c.

SECTION – IV

Answer **any 2** questions among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) State and prove Archimedean property.

    b) If S is a subset of $\mathbb{R}$ that contains at least two points and has the property if $x, y \in S$ with $x < y$, then $[x, y] \subseteq S$, prove that S is an interval.

33. a) State and prove Monotone Convergence Theorem.

    b) State and prove Bolzano – Weierstrass Theorem.

34. a) Let $p > 0$. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

    b) State and prove ratio test.

35. a) State and prove the sequential criterion for limits of a function.

    b) Show that $\lim_{x \to 0} \sin \frac{1}{x}$ does not exist.
