(Pages : 4) **M – 2335**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1231.1 : RANDOM VARIABLES AND ANALYSIS OF BIVARIATE DATA

(2014 – 17 Admissions)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions each carries **1** mark.

1. Define a discrete random variable.
2. What are the properties of the probability distribution function.
3. If X is a continuous random variable and A any real number what is the $P(X = A)$.
4. Define characteristic function.
5. Give the expression for Spearman's Rank Correlation Coefficient.
6. Define expectation of a continuous random variable.
7. Write the expression for variance in terms of expectation.
8. Given the distribution of the random variable X. Find the distribution of $Y = 2x + 3$.

X	1	2	3	4
$P(X = x)$	1/3	1/4	1/6	1/4

P.T.O.

9. What do you mean by normal equation?

10. If $Mx(t)$ is the mgf of a random variable X, find the mgf of $Y=aX+b$, a and $b \in R$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions, each carries 2 marks.

11. Can the given function be a probability density function
$$f(x)=\begin{cases} 2x & if \quad 0 \le x<1 \\ 4-2x & if \quad 1< x <2 \\ 0 & \text{otherwise} \end{cases}$$

12. Find C if $f(x)=C(2/3)^x$, $x=1, 2, 3.,,$ is a probability density function.

13. If the distribution function of a random variable is $F(x)=(3x^2-x^3)/4$ for $0 \le x \le 2$, find the pdf.

14. For a random variable X with pdf. find mean $f(x)=\frac{3}{4}x(2-x), 0<x<2$.

15. Show that the expectation of the product of two independent random variables is the product of their expectations.

16. Prove that the mgf of the sum of two independent random variables is the product of their respective mgf's.

17. Define conditional variance.

18. Distinguish between univariate and bivariate data.

19. Find standard deviation of Y if cov $(X, y) = 4.8$, $V(x) = 9$ and correlation between X and Y is 0.6.

20. How can you use scatter diagram to obtain an idea of the extend and nature of the correlation coefficient.

21. Comment on the statement $r_{xy} = 0$ implies X and Y are independent, where r_{xy} is the correlation coefficient.

22. Establish the relation between regression coefficients and correlation coefficient.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions, each carries **4** marks.

23. A random variable X has the following probability function.

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find (a) value of k (b) $P(X \geq 6)$ (c) $P(5 < X < 9)$.

24. Define (a) joint probability mass function (b) joint density function (c) conditional probability mass function (d) marginal probability mass function.

25. State and prove addition theorem of expectation.

26. The joint probability density function $f(x, y) = \begin{cases} 2 - x - y & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$.

Find marginal probability density functions of x and y.

27. State and prove Cauchy-Schwartz inequality.

28. Explain the principle of least squares.

29. If X is a random variable with moment generating function $M_x(t)$ and $Z = (X - a)/b$, a, b constants. Find moment generating function $M_z(t)$ of Z.

30. Let $P(X = k) = q^{k-1} p$, $k = 1, 2, 3, ..$ find the mgf of X.

31. Find correlation coefficient and mean of X and Y if the regression equations are $8x - 10y + 66 = 0$ and $40x - 18y = 214$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions, each carries **15** marks.

32. The joint probability distribution of random variables X and Y is $P(X=0, Y=1)=1/3$ $P(X=1, Y=-1)=1/3$ $P(X=1, Y=1)=1/3$ find

    (a) Marginal distribution of X and Y

    (b) Conditional distribution of X given Y and Y given X

    (c) Conditional distribution of X given Y = 1 and conditional distribution of Y given X = 0.

33. (a) Show that the expected value of X is equal to the expectation of the conditional expectation of X given Y is $E(x)=E(E(X/Y))$.

    (b) If $f(x, y)=\begin{cases} 2, & 0<x<y<1 \\ 0, & \text{elsewhere} \end{cases}$ find the correlation between X and Y.

34. (a) What is a scatter plot and what is meant by curve fitting?

    (b) Fit a curve of the form $Y=ab^X$ to the following data

X	1	2	3	4	5	6	7	8
Y	1	1.2	1.8	2.5	3.6	4.7	6.6	9.1

35. Following data gives the marks in 2 subjects in an examination. Mean mark in A = 39.5 Mean mark in B = 47.5, standard deviation of marks in A = 10.8, Standard deviation of marks in B = 16.8, correlation coefficient between marks in A and marks in B is 0.42

    (a) Draw the two lines of Regression

    (b) Explain why there are two Regression equations.

    (c) Give the estimate of marks in B for candidate who secured 50 marks in A.

(2 × 15 = 30 Marks)