(Pages : 4) **G – 2406**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme Under CBCSS

Complementary Course for Mathematics

ST 1231.1 — PROBABILITY THEORY

(2013 Admission)

Time : 3 Hours Max. Marks : 80

Use of Scientific Calculator permitted

SECTION – A

Answer **all** the questions, each carrying 1 mark.

1. If $B \subset A$, find $P(A \mid B)$.

2. If A and B are any two events $P(A \cap B)$ is

   (a) $= P(A)P(B)$ (b) $\geq P(A)P(B)$

   (c) $\leq P(A)P(B)$ (d) any of the above

3. State the multiplication theorem of probability for two events.

4. The relative frequency definition of probability is based on the principle of ________________.

5. Give the sample space corresponding to the random experiment of tossing three coins simultaneously.

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P.T.O.

6. For two dependent random variables X and Y, $E(X \mid Y = y)$ is a

(a) constant (b) function of X only

(c) function of y only (d) function of both X and Y

If a Continuous random variable X has a symmetric probability distribution over (–1, 1), the median of X is ____________

8. Define mathematical expectation of a discrete random variable.

9. What do you mean by probability density function of a continuous random variable X?

10. Can the probability density function of a continuous random variable assume a value greater than 1.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions, each carrying 2 marks.

11. If $A \subset B$, show that $P(A \mid B) \geq P(A)$.

12. Can $F(x) = \frac{x}{2}, 0 < x < 2$ be the cumulative distribution function of a random variable. Justify.

13. If a random variable X has pdf $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & otherwise \end{cases}$, find $P(X < 0.5)$.

14. If X has pmf as follows, find the pmf of 2X + 1

X :	0	1	2
P (X=x) :	0.3	0.4	0.3

15. If X is the number of tails and Y is the number of heads obtained when a coin is tossed 10 times, can you say whether X and Y are independent. Justify.

16. What do you mean by marginal distributions?

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17. If a fair coin is tossed till head occurs and X is the number of tosses required, find the probability distribution of X.

18. Let A and B be events such that $A \subset B$ and C be any other event. Check whether $P(A \mid C) \leq P(B \mid C)$.

19. What are the limitations of classical definition of probability?

20. Show that E(XY) = E(X)E(Y) if X and Y are independent random variables.

21. If (X, Y) has joint pdf $f(x, y) = \begin{cases} ke^{-(2x+3y)} & , \quad x > 0, y > 0 \\ 0 & , \quad othewise \end{cases}$. Find K.

22. If X has pmf $f(x) = \begin{cases} \frac{x}{10} & , \quad x = 1, 2, 3, 4 \\ 0 & , \quad otherwise \end{cases}$. Find $E(X)$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each carrying 4 marks.

23. Show that characteristic function of a random variable always exists.

24. If two persons A and B throw a fair dice in succession till '6 ' turns up and the first person to get a 6 wins the game, find A's chance of winning the game.

25. If X is the number of heads obtained when 4 fair coins are tossed simultaneously, obtain the pmf of X and find the mode of X.

26. Show that $E[E(X \mid Y)] = E(X)$.

27. What are the characteristics of a random experiment?

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28. If X is the number that turns up when an unbiased dice is thrown, find the mean and variance of X.

29. State and prove Bayes theorem.

30. In a certain group of mathematicians, 60 percent have insufficient background of Modern Algebra, 50 percent have inadequate knowledge of Mathematical Statistics and 70 percent are in either one or both of the two categories. What is the percentage of people who know Mathematical Statistics among those who have a sufficien background of Modern Algebra?

31. Let $p(x)$ be the probability function of a discrete random variable X which assumes the values x_1, x_2, x_3, x_4 such that $2p(x_1) = 3p(x_2) = p(x_3) = 5p(x_4)$. Find probability distribution of X.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each carrying 15 marks.

32. If X is the sum of the numbers obtained and Y is the maximum of the numbers Y obtained when an unbiased dice is thrown two times, find the joint pmf of X and Y. Also find the marginal distributions.

33. A, B and C are three urns which contain (2 white, 1 black), (3 white,2 black) and (2 white, 2 black) balls respectively. One ball is drawn from urn A and put into urn B, then a ball is drawn from urn B and put into urn C. Then a ball is drawn from urn C. Find the probability that the ball drawn is black.

34. A fair coin is tossed 4 times. Let X denote the number of times a head is followed immediately by a tail. Find the probability distribution, mean and variance of X. Also find the probability distribution if the coin is biased with probability of head 'p'=1/3.

35. Let $f(x,y)=\begin{cases} kxy & , \quad 0<x<y<1 \\ 0 & , \quad otherwise \end{cases}$. Find k, $E(Y \mid X=x), E(XY \mid X=x)$.

(2 × 15 = 30 Marks)

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