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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, August 2018
First Degree Programme under CBCSS
COMPLEMENTARY COURSE FOR MATHEMATICS
ST 1231.1 : Probability Theory
(2013 Admn.)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** the questions. **Each** carries **one** mark.

1. Give an example of exhaustive events.
2. State any one limitation of frequency definition of probability.
3. Who introduced classical definition of probability ?
4. State addition theorem of probability.
5. If $P(A|B) = \frac{1}{4}$ and $P(B|A) = \frac{2}{5}$, then find $\frac{P(A)}{P(B)}$.
6. State the properties of probability density function of a continuous random variable.
7. If X is the number of heads obtained when three balanced coins are tossed together, find the probability distribution of X.
8. Give the domain and range of probability mass function of a discrete random variable.
9. Sketch the graph of CDF of a continuous random variable.
10. What do you mean by independent random variables ? **(1×10=10 Marks)**

P.T.O.

SECTION – B

Answer **any eight** questions. **Each** carries **two** marks.

11. For a random experiment, the sample space S = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 2, 3} and B = {3, 4, 5, 6}. Write the events (i) $(A \cap B)^c$ (ii) $A \cap B^c$.

12. Suppose you are given an unbiased die and you are asked to find the probability of the event "an odd number turns up when the die is rolled". Explain how do you obtain this probability using the empirical and classical definitions of probability.

13. Let S = {1, 2, 3, 4, 5, 6} be a sample space and let A = {1, 2, 3, 4}, $P(A) = \frac{1}{3}$. Write down the probability space.

14. Three unbiased dice are rolled. What is the probability that they all show different faces ?

15. Justify the term posterior probability in Baye's formula.

16. The pdf of X is given by $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \theta > 0, x > 0$ and 0 otherwise. Obtain the CDF of X and hence find $P(X \leq 3)$.

17. Show that the mathematical expectation of the product of a number of independent random variables is equal to the product of their expectations.

18. What do you mean by characteristic function of a random variable ?

19. Let X be a discrete random variable with the following mass function $f(x) = \frac{kx}{10}$; x = 0, 1, 2, 3, 4 and 0 otherwise. Determine the value of k and obtain P[X > 2].

20. Let X be a continuous random variable with the following density function $f(x) = 3x^2$; 0 < x < 1 and 0 elsewhere. Find the distribution function of X.

21. The total lifetime (in years) of one-year-old dogs of a certain breed is a random variable whose CDF is given by

$$F(x) = \begin{cases} 0 & X \leq 1 \\ 1 - \frac{1}{x^2} & X > 1 \end{cases}$$

Find the probability that such a one-year-old dog will live 8 or more years.

22. If the joint density function of (X, Y) is given by

$f(x, y) = x + y, 0 \le x \le 1, 0 \le y \le 1$

Find Cov(X, Y). **(2×8=16 Marks)**

SECTION – C

Answer **any six** questions. **Each** carries **four** marks.

23. If A_1, A_2, A_3 are mutually exclusive and exhaustive events show that $B_1 = A_1$, $B_2 = A_1^c A_2$ and $B_3 = A_1^c A_2^c A_3$ are mutually exclusive and exhaustive.

24. State and prove multiplication theorem of probability.

25. In a factory, machines A and B are producing items of the same type. Of this production, machines A and B produce 5% and 10% defective springs respectively. Machines A and B produce 40% and 20% of the total output of the factory. One item is selected at random and it is found to be defective. What is the probability that the defective item is produced by machine A ?

26. Two unbiased dice are rolled together. Let X denotes the sum of face values of the two dice. Find the mean of X.

27. Suppose that a cancer diagnostic test is 95% accurate both on those that do and those that do not have the disease. If 0.4% of the population have cancer, compute the probability that a tested person chosen randomly from the population, has cancer, given that his/her test result indicates so.

28. Let the joint pmf of (X, Y) be $f(x,y) = \frac{x+y}{21}$; x = 1, 2, 3; y = 1, 2 find E(X) and E(Y).

29. Define moment generating function and discuss its uses.

30. What is $P[X > m + n \mid X > m]$, if X has pmf $f(x) = p(1-p)^x$; $0 < p < 1$, x = 0, 1, 2, .. and 0 elsewhere ?

31. Find the Distribution of $Y = -\log X$, if X has pdf $f(x) = 1$; $0 < x < 1$ and 0 elsewhere. **(4×6=24 Marks)**

SECTION – D

Answer **any two** questions. **Each** carries **fifteen** marks.

32. A certain product was found to have two types of defects. Suppose that the probability that a randomly chosen item has only a type-1 defect is 0.2 and the probability that it has only type-2 defect is 0.3. Also, the probability that it has both defects is 0.1. Find the probabilities of the following events :
    i) It has either a type-1 defect or type-2 defect or both
    ii) It does not have either of the defects
    iii) It has type-1 defect but not type-2 defect
    iv) It has exactly one of the defects.

33. Suppose that, for adults under age 50, we are interested in comparing sleep disorders (A) between males (M) and females (F). It is known that 70% of males and 40% of females have sleep disorders. Compute $P(M|A)$ and $P(F|A)$.

34. If X and Y have the joint density function $f(x) = \frac{1}{27}(2x + y)$; $x, y = 0, 1, 2$, find the conditional distribution of Y for given $X = x$.

35. If $f(x, y) = 8xy$; $0 < x < y < 1$ and 0 elsewhere, find the conditional densities and $E(X|Y = y)$ and $E(Y|X = x)$. **(15×2=30 Marks)**