21.02.22

(Pages : 4) **M – 2340**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Statistics

Complementary Course for Mathematics

ST 1231.1 : PROBABILITY AND RANDOM VARIABLES

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define sample space.

2. Define statistical regularity.

3. Prove that $P(\phi)=0$.

4. If A and B are independent, then what is $P(A/B)$ and $P(B/A)$?

5. State the multiplication theorem of probability.

6. Define compound probability.

7. Give the definition of the distribution function of a two dimensional random vector.

8. Express variance in terms of conditional variance and conditional expectation.

P.T.O.

9. State the linearity property of expectation.

10. Define moment generating function associated with a random variable.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Distinguish between mutually exclusive and exhaustive events with examples.

12. For any two events A and B in the same sample space prove that

    $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$.

13. Give the classical definition of probability and mention its drawbacks.

14. Define probability as a 'measure', explaining the term 'measure'.

15. What do you mean a probability space?

16. State the law of total probability.

17. If A and B are independent events then prove that A and B^C are independent and A^c and B are independent.

18. If A, B and C are mutually independent, then prove that $A \cup B$ and C are independent.

19. If the joint pdf of X and Y is $p(x,y) = \frac{1}{22}(2x+3y), x = 0,1$ and $y = 1,2$ what is the joint distribution function?

20. Find the conditional pdf of X/Y, if the joint pdf of (X,Y) is $f(x,y) = 2 - x - y, 0 < x, y < 1$.

21. When do you say that two random variables are stochastically independent?

22. Define the concept of Transformation of one dimensional random variables.

23. Show that $E(X) = E(E(X/Y))$, with usual notations.

24. Write the expression for the r^{th} and s^{th} product moment about the origin of the bivariate random vector (X, Y).

25. Give the Cauchy-Schwartz inequality.

26. Show that $\phi_X(t)$ and $\phi_X(-t)$ are complex conjugate functions, where $\phi_X(t)$ is the characteristic function of the random variable X.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

27. Give the axiomatic definition of probability.

28. Tom speaks truth in 30 percent cases and Jerry speaks truth in 25 percent cases. What is the probability that they would contradict each other?

29. Discuss the frequency/statistical definition of probability. Using this definition, prove that $P(A^C) = 1 - P(A)$.

30. Suppose A, B and C are events such that $P(A) = P(B) = P(C) = 1/4$, $P(A \cap B) = 0 = P(B \cap C)$ and $P(A \cap C) = \frac{1}{8}$. Evaluate $P(A \cup B \cup C)$.

31. Each of the three guns has a probability 0.4 of hitting a target. What is the probability that (a) all will hit the target and (b) at least one will hit the target?

32. If $p(x) = c\left(\frac{2}{3}\right)^x, x = 1,2,....$ is a probability mass function,(a) find c and (b) $P(1 < X < 3)$.

33. The joint pdf of a two dimensional random variable is $f(x, y) = 2, 0 < x < 1; 0 < y < x$.

    Then (a) find the marginal pdfs of X and Y and (b) check whether X and Y are independent

34. A continuous random variable X has the pdf $f(x) = Ae^{-x/\theta}, x > 0$. (a) Find A and (b) for any two positive integers s and t prove that $P(X > s + t / X > t) = P(X > s)$.

35. If $f(x)=1,, 0<x<1$, find the pdf of $Y=-2\log X$.

36. A balanced die is rolled. If a person receives Rs. 10 when he gets an even number and loses Rs. 8 when he gets an odd number, how much money can he expect on an average in the long run.?

37. Let $f(x,y)=8xy, 0<x<y<1$. Find (a) $E(Y/X)$ and (b) $V(Y/X)$.

38. Mention the important properties of a characteristic function.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

39. From a group of 8 children including 5 boys and 3 girls, three children are selected at random. What is the probability that (a) No girl (b) only one girl (c) one particular girl (d) at least one girl (e) More girls than boys are selected?

40. Prove or disprove: Mutual independence of three events implies pairwise independent. Is the converse true? Justify your claim.

41. (a) State and prove Baye's theorem.

    (b) Three identical boxes contain two balls each. One has both red, one has one red and one black and the third has two black balls. A person chooses a box at random and takes out a ball. If the ball is red and the probability that the other ball in the box is also red.

42. Let $f(x,y)=\frac{1}{72}(2x+3y), x=0,1,2$ & $y=1,2,3$ be the joint pdf of (X,Y). Then find (a) the distribution of $X+Y$, (b) the conditional distribution of $X/X+Y=3$ and (c) examine whether X and Y are independent or not.

43. Find the moment generating function of X with pdf $f(x) = \frac{1}{\theta}, 0<x<\theta$ and 0 elsewhere. Also compute the mean and the first 4 central moments.

44. Let X and Y have the joint pdf $f(xy) = \frac{x+2y}{18}, x=1,2; y=1,2$. Find the coefficient of correlation between X and Y.

(2 × 15 = 30 Marks)