(Pages : 6) **J – 2701**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2020

First Degree Programme Under CBCSS

Complementary Course for Mathematics

ST 1231.1 — PROBABILITY AND RANDOM VARIABLES

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

Instructions : Scientific Calculators and Mathematical/Statistical tables are permitted to use.

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define the sample space of a random experiment and give an example.

2. In a population of N balls, there are M white balls and remaining are black balls. What is the probability that the group so selected will contain exactly m white balls, if n balls are selected from the population?

3. Let A and B be any two events defined on a sample space then prove that $P(A \cap B^c) = P(A) - P(A \cap B)$ where A^c is the complement of A.

4. If five cards are selected from a pack of 52 cards, find the probability of getting at least three spade cards.

5. If the probability mass function of a discrete random variable is given by $f(x) = \frac{1}{2^x}$ where $x = 1,2,3,\ldots$ find moment generating function of X.

6. Let X be a random variable with distribution function $F(x) = 1 - e^{-\theta x}$; where $x \geq 0$; $\theta \geq 0$ find probability density function of X.

7. If x is a random variable with following probability mass function

x:	−1	0	1	2
p(x):	3/8	1/8	1/8	3/8

Find $E(X^2)$.

8. If X is a random variable then what is the value of $E[X(X-1)] - E(X)E(X-1)$ where $E(X)$ is the expectation of X.

9. If $M_X(t) = \frac{2}{2-e^t}$ is the moment generating function of X, find the moment generating function of $2X+1$.

10. Define conditional expectation of a random variable X given Y.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Explain concept Statistical regularity.

12. Three unbiased coins are tossed together find the probability distribution of number of heads and sketch the graph of the function.

13. State and prove addition theorem on probability.

14. Let $P(A) = 0.4$ and $P(A \cup B) = 0.6$ for what values of $P(B)$ are A and B independent events?

15. If A and B are any two independent events verify the independence of A^c and B^c.

16. A random variable X has the probability density function $f(x) = \begin{cases} 1/4 & -2 < x < 2 \\ 0 & otherwise \end{cases}$. Find (i) distribution function of X (ii) $P(|x| > 1)$.

17. For a random variable X, probability mass function is given by $f(x) = \begin{cases} \frac{x}{k}, & x = 1,2,3,4,5 \\ 0, & otherwise \end{cases}$. Find K and hence obtain $P(X \geq 2)$. What is the distribution of $Y = (X - 3)^2$.

18. A random variable X has the following probability density function $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$. Find the probability distribution function of X.

19. If $M_X(t) = \left(\frac{2}{3} + \frac{1}{3}e^t\right)^4$ is the moment generating function of X, what is $E(X)$?

20. If $\Phi_X(t)$ is the characteristic function of X derive the characteristic function of $Y = aX + b$ where a and b are constants.

21. Let X be a non negative continuous random variable with probability density function $f(x)$ then obtain the probability density function of $Y = X^2$

22. If $f(x, y) = K(x + 2y)$, $x = 0,1,2$; $y = 1,2,3$ is the joint probability function of X and Y then obtain the value of K. Also find $E(XY)$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$ then find the following probabilities (i) at least one of the two events (ii) exactly one of the two events (iii) none of the events occur (iv) $P(A / B^c)$.

24. Prove that $P(A \cup B / C) = P(A / C) + P(B / C) - P(A \cap B / C)$.

25. State and prove Bayes' theorem.

26. Prove that $E(aX + bY) = aE(X) + bE(Y)$ where a and b are constants.

27. Let X has the moment generating function $M_X(t) = (1 - t)^{-2}$ for $|t| < 1$ then what is the mean and variance of x?

28. Show that $(E(XY))^2 \leq E(X^2)E(Y^2)$.

29. Two random variables have joint probability mass function $P(X = 0, Y = 0) = 2/9$, $P(X = 0, Y = 1) = 1/9$, $P(X = 1, Y = 0) = 1/9$, and $P(X = 1, Y = 1) = 5/9$). Find the value of $E(X + Y)$. Also examine independence of random variables X and Y.

30. The joint probability mass function of (X,Y) is given by $f(x, y) = \frac{1}{16}$ for $(x, y) = (-3, -5), (-1, -1), (1, 1)$ and $(3,5)$. Find the covariance between X and Y.

31. If $V(X)$ is the variance of a random variable then show that it can be represented as $V(X) = E(V(X / Y)) + V(E(X / Y))$ where $E(X/Y)$ and $V(X/Y)$ are conditional mean and conditional variance of X given Y.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (i) Explain the concept of pair wise independence and mutual independence between n events. Give examples.

(ii) In a factory total items are produced by 4 machines in the proportion 4:7:8:6 and of their output, respectively 8%, 3%, 4% and 5% are defective items. An item is selected at random then find the chance that

(a) it is a non defective

(b) It is produced by the third machine if the selected was a defective.

33. (i) A box contains 2^n tickets of which $\binom{n}{i}$ tickets bear the number $i = 0,1,2,\ldots n$.

A group of m tickets are drawn, what is the expectation of the sum of their numbers.

(ii) If X is a random variable with probability mass function $f(x) = \frac{1}{n}$, $x = 1,2,\ldots n$ then find mean, variance and moment generating function of X.

34. (i) A random variable X has the cumulative distribution

$$F(x) = \begin{cases} 0 & \text{if} \quad x \le 0 \\ \frac{1}{2}x & \text{if} \quad 0 < x < 1 \\ x - 1/2 & \text{if} \quad 1 \le x < 3/2 \\ 1 & \text{if} \quad x \ge 3/2 \end{cases}$$

. Find probability density function of X.

Also obtain $P(X > 1/2)$, $P(X \le 5/4)$ and $P(X = 5/4)$. Sketch the graph of both distribution and density function of x.

(ii) Let X and Y are jointly distributed with probability density function $f(x, y) = \begin{cases} e^{-(x+y)} & \text{if} \quad 0 < x < \infty,\ 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$. Show that X and Y are independent random variables.

35. (i) Let X and Y are two random variables having joint probability mass function $f(x, y) = \frac{1}{72}(2x + 3y),\ x = 0,1,2;\ y = 1,2,3$

Find (a) marginal probability mass functions of X and Y

(b) conditional distribution of X given $Y = 1$,

(c) conditional mean of X given $Y = 1$.

(ii) Let X and Y have the joint probability density function $f(x, y) = k, x, y, 0 < x < y < 1$, find k and marginal probability density functions of X and Y.

(2 × 15 = 30 Marks)
