(Pages : 4) **M – 2336**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Foundation Course – II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. What is antecedent in a conditional statement?

2. Define tautology.

3. Give the negation of "*S* is compact and convex."

4. Express the statement "If x is greater than 1, then x^2 is greater than 1" using quantifiers.

5. Define half-open Intervals.

6. Let L be the length of the curve $x = \ln t, y = \sin t (1 \leq t \leq \pi)$. Find an integral expression for L.

7. Write the formula for the area A of the region R enclosed by the polar curve $r = f(\theta)(\alpha \leq \theta \leq \beta)$ and the lines $\theta = \alpha$ and $\theta = \beta$.

P.T.O.

8. If $v = \langle -2, 0, 1 \rangle$ and $w = \langle 3, 5, -4 \rangle$, the find $w - 2v$.

9. Express $u \times v$ as a determinant.

10. Define trace of a surface in a plane.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the questions **11** to **22**. These questions carry **2** marks each.

11. Draw the truth table of the biconditional statement.

12. What are the two types of quantifiers?

13. Prove that $(\sim p \Rightarrow c) \Leftrightarrow p$.

14. Prove that $A \cap (U \setminus B) = A \setminus B$, where A and B be any two sets and U is the universal set.

15. Prove that $(a,b) = (c,d)$ if and only if $a = c$ and $b = d$.

16. Define equivalence relation. Give an example.

17. Eliminate t and establish a relation between x and y, given $x = 2t - 3$, $y = 6t - 7$.

18. Find the slope of the tangent line to the unit circle $x = \cos t$, $y = \sin t$ at the point $t = \frac{\pi}{6}$.

19. Find the circumference of a circle having radius 'a' whose parametric equation is $x = a \cos t$, $y = a \sin t$.

20. Find the center and radius of the sphere $x^2 + y^2 + z^2 - 2x - 4y + 8z + 17 = 0$.

21. Find a vector that is orthogonal to both of the vectors $u = \langle 2, -1, 3 \rangle$ and $v = \langle -7, 2, -1 \rangle$.

22. Calculate the scalar triple product $u.(v \times w)$ of the vectors $u = 3i - 2j - 5k$, $v = i + 4j - 4k, w = 3j + 2k$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions 23 to 31. These questions carry **4** marks each.

23. Prove that $p \Rightarrow q$ and $\sim q \Rightarrow \sim p$ are logically equivalent.

24. Prove that "For every $\varepsilon > 0$ there exists $\delta > 0$ such that $1-\delta < x < 1+\delta \Rightarrow 5-\varepsilon < 2x+3 < 5+\varepsilon$".

25. Let $f(x)$ be an integrable function. If $\int_0^1 f(x) \neq 0$, then prove that there exists an x in $[0,1]$ such that $f(x) \neq 0$.

26. Sketch the trajectory over the time interval $0 \le t \le 10$ of the particle whose parametric equations of motion are $x = t - 3\sin t, y = 4 - 3\cos t$.

27. Without eliminating the parameter, find dy/dx and d^2y/dx^2 at (1, 1) and (1, –1) on the semicubical parabola given by the parametric equations $x = t^2, y = t^3$.

28. Sketch the graph of the equation $r = \sin\theta$ in polar coordinates by plotting points.

29. If u and v are nonzero vectors in 2-space or 3-space, and if θ is the angle between them, then prove that $\cos\theta = \dfrac{u.v}{\|u\|\,\|v\|}$.

30. Let $v = <2, 3>$, $e_1 = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, e_2 = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$. Find the scalar components of v along e_1 and e_2 and the vector components of v along e_1 and e_2.

31. Find the orthogonal projection of $v = j + j + k$ on $b = 2i + 2j$, and then find the vector component of v orthogonal to b.

(6 × 4 = 24 Marks)