(Pages : 4) **M – 2334**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Foundation Course II

MM 1221 – FOUNDATIONS OF MATHEMATICS

(2014-2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. Each question carries **1** mark.

Answer in one word to a maximum of **2** sentence.

1. Find the order of 2 modulo 7.

2. Find the inverse of $[3]$ in $Z/5Z$.

3. In which congruence class in $Z/3Z$ is 3295.

4. Find the x-coordinates of all inflection points of $3x^4 - 4x^3$.

5. Verify that $\phi(ab) = \phi(a).\ \phi(b)$ when $a = 4,\ b = 7$.

6. Evaluate $\int \frac{\sec^2 x}{1 + \tan x}\, dx$.

P.T.O.

7. Find $\int_1^5 f(x)\,dx$ if $\int_0^1 f(x)\,dx = -2$ and $\int_0^5 f(x)\,dx = 1$.

8. Find the average value of $f(x) = x^2$ over $[0, 2]$.

9. Find the polar co-ordinates of the point whose rectangular co-ordinates are $(-\sqrt{3},1)$ satisfying $r \geq 0$ and $0 \leq \theta \leq 2\pi$.

10. Find the eccentricity and the distance from the pole to the directrix of the conic $r = \dfrac{3}{2 + \sin\theta}$.

SECTION – II

Answer **any eight** questions (**11-22**). Each question carries **2** marks.

11. In $Z/14Z$, solve $[6]\,X = [10]$.

12. In $Z/13Z$, find the inverse of $[2], [4], [5]$ and $[7]$.

13. Examine whether $\{1, 3, 5, 7, 9, 11, 13\}$ is a complete set of representatives for $Z/7Z$.

14. Write down the addition table for arithmetic modulo 5.

15. If p is prime , prove that $\phi(p) = p - 1$.

16. Find remainder when 2^{1000} is divided by 17.

17. Evaluate $\int_0^3 f(2x)\,dx$ if $\int_0^6 f(2x)\,dx = 3$.

18. Evaluate $\int_{-1}^1 x|x|\,dx$.

19. Find the displacement and distance travelled during the time interval $0 \leq t \leq 3$ if the velocity function is $v(t) = t^2 - 2t$.

20. Find area of the surface that is generated by revolving the portion of the curve $y^2 = x$ from origin to the point where $x = 2$ about the x-axis.

21. Evaluate $\int_{x/3}^{\pi/2} \sin\theta\left(1 - 4\cos^2\theta\right) d\theta$.

22. Find the entire area within the cardioids $r = 2 + 2\cos\theta$.

SECTION – III

Answer **any six** questions (**23-31**). Each question carries **4** marks.

23. Find the orders of the non zero elements of $Z/5Z$.

24. In Z/mZ show that $[a]$ is a unit if and only if $(a, m) = 1$.

25. State Fermat's theorem and verify if for $a = 3$ and $p = 7$.

26. Find the exact arc length of the curve $y = 3x^{3/2} - 1$ from $x = 0$ to $x = 1$.

27. Evaluate without finding anti-derivative :

(a) $\int_{-1}^{2} (x + 2)\, dx$

(b) $\int_{0}^{2} \sqrt{4 - x^2}\, dx$.

28. Suppose that a curve $y = f(x)$ in the xy– plane has the property that at each point (x, y) on the curve, the tangent line has slope x^2. Find an equation for the curve given that it passes through the point $(2, 1)$.

29. Find the volume of the solid generated by the revolution of the loop of the curve $y^2 = x^4(x+2)$ about the x-axis.

30. Find the surface area generated by revolving one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, $0 < \theta < 2\pi$ about the x-axis.

31. Find the relative extrema of $f(x, y) = x^3y^2(12 - x - y)$.

SECTION – IV

Answer **any two** questions (**32-35**). Each question carries **15** marks.

32. (a) Prove that, if n is positive and a is prime to $na^{\phi(n)} \equiv 1 \pmod{n}$. Verify the result for $m = 14$ and $a = 5$.

    (b) If e is the order of a modulo m and $a^f \equiv 1$ (mod m), prove that e divides f.

33. (a) Find the order of $[2]$ in Z/mZ where

    (i) $m = 11$

    (ii) $m = 31$

    (b) Show that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is an integer for all n.

34. (a) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^3 - 3x^2 + 2x$ over $[0, 1]$ is revolved about the y-axis.

    (b) Find the area enclosed by the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

35. (a) State the horizontal line test. Use the horizontal line test to show that $f(x) = x^2$ has no inverse but that $f(x) = x^3$ does.

    (b) Sketch the graph of the equation $r = \sin\theta$ in polar coordinates.