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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, August 2018
First Degree Programme under CBCSS
MATHEMATICS
Foundation Course – II
MM 1221 : Foundations of Mathematics
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. They carry **1** mark **each**.

1. Find all the numbers a, $0 \leq a < 36$ with $8a \equiv 0 \pmod{36}$.

2. Give a complete set of representatives for $\mathbb{Z} / 7\mathbb{Z}$.

3. In which congruence class in $\mathbb{Z} / 5\mathbb{Z}$ is 2017 ?

4. Define Euler's Phi function.

5. Find the critical points of $f(x) = 4x^4 - 16x^2 + 17$.

6. Define an anti derivatives of a function f on a given open interval (a, b).

7. State the fundamental theorem of calculus and obtain $\int_2^3 x\,dx$.

8. State L Hospital's rule.

9. Find the polar coordinates of the point whose rectangular coordinates (– 7, 0) satisfies $r \geq 0$ and $0 \leq \theta \leq 2\pi$.

10. Express the Cartesian equation $x^2 + y^2 = 14$ in polar coordinates.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. What are the units of $\mathbb{Z}/9\mathbb{Z}$? Also find the elements that are not units.

12. Find the inverses, of 1, 2, 3, 4 in $\mathbb{Z}/5\mathbb{Z}$.

13. Solve $[6]X = [3]$ in $\mathbb{Z}/11\mathbb{Z}$.

14. Verify that $\phi(ab) = \phi(a)\,\phi(b)$ for $a = 5$, $b = 6$.

15. Write down the addition and multiplication tables for arithmetic modulo 4.

16. Find the remainder when 2^{1000} is divided by 17.

17. Evaluate the double integral $\iint_R x\sqrt{1-x^2}\ dA$ over the rectangular region $R = \{(x, y) : 0 \le x \le 1, 2 \le y \le 3\}$.

18. Evaluate the integral $\int \frac{x^7}{\sqrt{x^4+2}}\,dx$ by making the substitution $u = x^4 + 2$.

19. A particle u moves along an s-axis, with a velocity $v(t) = 3t^2 - 2t$, $s(0) = 1$. Find the position function of the particle.

20. Evaluate $\int \cos^4 3t \sin 3t\, dt$.

21. Sketch the region whose signed area is represented by the definite integral $\int_0^3 x\, dx$ and evaluate the integral using an appropriate formula from geometry.

22. Find the area enclosed with in the curve $r = 4(1 + \cos\theta)$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Define the order of a modulo m and find the order of the nonzero elements of $\mathbb{Z}/5\mathbb{Z}$.

24. If a and m are relatively prime, then prove that $a^t \equiv 1 \pmod m$ for some $t, 1 \leq t < m$.

25. Prove that [a] is a unit of $\mathbb{Z} / m\mathbb{Z}$ if and only if $(a, m) = 1$.

26. A projectile is launched vertically upward from ground level with an initial velocity of 112 ft/sc. Given the position and velocity function for a particle in free-fall motion as $s(t) = s_0 + v_0(t) - \frac{1}{2}gt^2$ and $v(t) = v_0 - gt$.

   i) Find the velocity at $t = 3$ seconds and $t = 5$ seconds

   ii) How high will the projectile rise ?

   iii) Find the speed of the projectile when it hits the ground.

27. Find the area enclosed between the parabola $y = x^2$ and the straight line $2x - y + 3 = 0$.

28. Define the average value of a function f on [a, b]. Suppose that the velocity function of a particle moving along a coordinate line is $v(t) = 3t^3 + 2$ find the average velocity of the particle over the time interval $1 \leq t \leq 4$ by integrating.

29. Find the volume of the solid that results when the region about the x-axis and below the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > 0, b > 0$ is revolved about the x-axis.

30. Show that the area of the surface of a sphere of radius r is $4\pi r^2$.

31. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10.

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) State and prove Fermat's theorem.

   b) Prove that if p is prime, then for any number a, divisible by p or not, $a^p \equiv a \pmod p$.

33. a) i) If $\varphi(m)$ is the number of units in $\mathbb{Z}/m\mathbb{Z}$, then prove that for any unit $[a]$ of $\mathbb{Z}/m\mathbb{Z}$, $[a]^{\varphi(m)} = 1$.

ii) If n is a prime number greater than 7, show that $n^6 \equiv 1 \pmod{504}$.

b) Show that $\{0, 2, 2^2, 2^3, \ldots, 2^{11}, 2^{12}\}$ is a complete set of representatives for $\mathbb{Z}/13\mathbb{Z}$.

34. a) Use cylindrical shells to find the volume of the cone generated when the triangle with vertices (0, 0), (0, r), (h, 0) where $r > 0$ and $h > 0$ is revolved about the x-axis.

b) Find the length of the arc of the catenary $= c\cosh\frac{x}{c}$ measured from its vertex to the point (x, y). Also show that $y^2 = c^2 + s^2$.

35. a) Derive the polar equations for the conic sections from their focus-directrix characterizations.

b) Sketch the graph of $r = \frac{6}{2+\cos\theta}$ in polar coordinates.