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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, March 2020

First Degree Programme under CBCSS

Complementary Course For Chemistry/ Polymer Chemistry

MM 1431.2 : MATHEMATICS IV (ABSTRACT ALGEBRA, AND LINEAR TRANSFORMATIONS)

(2014 – 17 Admission)

Time : 3 Hours Max. Marks : 80

PART – I

Answer **all** questions. Each question carries **1** mark.

1. Give an example of a non abelian group.

2. Write the generators of Z_6.

3. True or false. Every Cyclic group is abelian.

4. State whether true or false. Every field is also a ring.

5. Give an example of a ring that is not a field.

P.T.O.

6. Check whether the vectors $\begin{bmatrix}1\\-1\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}1\\2\end{bmatrix}$ are linearly independent.

7. Define a linear transformation.

8. If $T: R^2 \to R^2$ is defined by $T(x) = Ax$, Where $A = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$. Find the image of $\begin{bmatrix}1\\-3\end{bmatrix}$.

9. Find the standard matrix A for the dilation transformation $T_x = 2x$ for $x \in \mathbb{R}^3$.

10. Define a 1-1 mapping.

(10 × 1 = 10 Marks)

PART – II

Answer **any eight** questions. Each question carries **2** marks

11. Determine whether the binary operation * defined on Z by $a * b = a - b$ is commutative and whether * is associative.

12. If G is a group with binary operation *, show that a*b=a*c implies $b = c$ for all $a, b, c, \in G$.

13. Find the generators of Z_4.

14. Show that every cyclic group is abelian.

15. Define a ring. Give an example.

16. Find the number of generators of the cyclic subgroup of Z_{30} generated by 25.

17. Check whether the vectors $v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ are linearly independent.

18. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x) = Ax$, where $A = \begin{bmatrix} 1 & 0 & -3 \\ -3 & 1 & 6 \\ 2 & -2 & -1 \end{bmatrix}$. Find $T(b)$ where $b = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$.

19. Does the transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (x_1 - 2x_2,\ x_1 - 3,\ 2x_1 - 5x_2)$ linear?

20. Illustrate the transformation rotation. What is the matrix of this transformation?

21. Describe geometrically what T does to each vector x in R^2, where $T(x) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

22. Find the value of h such that $\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix}$ are linearly dependent.

(8 × 2 = 16 Marks)

PART – III

Answer **any six** questions. Each questions carries **4** marks

23. Define * on the set of positive rational numbers Q^+ by $a * b = \frac{ab}{2}$, show that $< Q^+, * >$ is a group.

24. Draw a subgroup diagram for the Klein-4 group.

25. Find all orders of the subgroups of Z_{20}.

26. Check whether the set $\{a + b\sqrt{2} / a, b \in Z\}$ with the usual addition and multiplication forms a ring. Is it a commutative ring?

27. Describe all units in the ring Z_5.

28. Determine whether the columns of the matrix $A = \begin{bmatrix} 0 & -3 & 9 \\ 2 & 1 & -7 \\ -1 & 4 & -5 \\ 1 & -4 & -2 \end{bmatrix}$ form a linearly independent set.

29. Find all x in R^4 that are mapped into the zero vector by the transformation $x \to Ax$ for the matrix $A = \begin{bmatrix} 1 & -3 & 5 & -5 \\ 0 & 1 & -3 & 5 \\ 2 & -4 & 4 & -4 \end{bmatrix}$.

30. Let T be the linear transformation whose standard matrix is $A=\begin{bmatrix}1 & -4 & 8 & 1\\ 0 & 2 & -1 & 3\\ 0 & 0 & 0 & 5\end{bmatrix}$ Does T map R^4 onto R^3? Is T a 1-1 mapping?

31. Let $T:R^2\rightarrow R^2$ be a linear transformation such that $T(x_1,x_2)=(x_1+x_2,4x_1+5x_2)$. Find x such that $T(x)=(3,8)$.

(6 × 4 = 24 Marks)

PART – IV

Answer **any two** questions. Each question carries **15** marks

32. (a) Find all subgroups of Z_{18} and give their subgroup diagram.

(b) Define permutation group. Find $\mu\sigma$ and σ^2 where $\mu=\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 3 & 1 & 4 & 5 & 6 & 2\end{pmatrix}$ and $\sigma=\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 2 & 4 & 1 & 3 & 6 & 5\end{pmatrix}$.

33. (a) Define a field. Give an example.

(b) Show that 2z, the set of all even integers is a ring under usual adition and multiplication. Is the ring commutative?

34. (a) Prove that $T:R^2\rightarrow R^2$ defined by $T(x_1,x_2)=(4x_1-2x_2,3|x_2|)$ is not linear.

(b) Find the standard matrix of the linear transformation that first reflects the points through the horizontal x_1 axis and then reflects the points through the line $x_2=x_1$.

35. (a) Show that as indexed set $s = \{v_1, v_2, \ldots v_p\}$ of two or more vectors is linearly independent if and only if atleast one of the vectors in s is a linear combination of the others.

(b) If the matrix given below determines a linear transformation T, find all x such that $T(x) = 0$.

$$\begin{bmatrix} -9 & -4 & -9 & 4 \\ 5 & -8 & -7 & 6 \\ 7 & 11 & 16 & -9 \\ 9 & -7 & -4 & 5 \end{bmatrix}$$

(2 × 15 = 30 Marks)
