(Pages : 4) **G – 2474**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme under CBCSS

Complementary Course

MM 1231.2 : MATHEMATICS — II – CALCULUS WITH APPLICATIONS IN CHEMISTRY - III [FOR CHEMISTRY AND POLYMER CHEMISTRY)

(2018 Admission)

Time : 3 Hours Max. Marks : 80

Section – I

All 10 questions are compulsory, each carries 1 mark each.

1. Find f_{xx} and f_{yy} of the function $f(x,y) = x^2y^2$.
2. If $w = e^x$ and $x = t^3$ then to find $\frac{dw}{dt}$.
3. Sum the series 1 to 100 inclusively.
4. Find $\sum_{n=0}^{n=9} ar^n$.
5. Define alternating series.
6. Define irrotational vector field.
7. If $F = e^{xyz}(\hat{i} + \hat{j} + \hat{k})$ then find $div F$.
8. Define the gradient vector field of a scalar field φ.

P.T.O.

9. Evaluate $\int_0^2 \int_0^x 2xy \, dx \, dy$.

10. If $x = r\cos t$ and $y = r\sin t$ then find $\frac{\partial(x,y)}{\partial(r,t)}$.

Section – II

Answer any **eight** questions, each carries 2 mark each.

11. Show that $xy^2dx + y^2x\,dy$ is exact differential.

12. If $f(x,y) = (x+y)^2$ then find $x f_x + y f_y$.

13. Given that $x(u) = 1 + au$ and $y(u) = bu^3$, where a and b are constants. Find rate of change of $f(x,y) = xe^{-y}$ with respect u.

14. State D'Alembert's ratio test on convergence of a series.

15. Using Integral test verify whether the series $\sum_{n=1}^{\infty} \frac{1}{\left(n - \frac{3}{2}\right)^2}$ convergent or not.

16. Show that the series $\sum_{n=1}^{\infty} \frac{1}{(n)^n}$ is convergent.

17. Find the Laplacian of $\varphi = x^3y^2z$.

18. If $\varphi = xyz$ then show that $\nabla \times \nabla_{\varphi} = 0$.

19. Find the length of the curve traced by the vector function. If $\vec{r}(t) = 2t\hat{i} + (3t)\hat{j} + (1-t)\hat{k}$ from $t = 0$ to $t = 2$.

20. Evaluate $\int_0^a \int_{y=0}^{\sqrt{a^2-x^2}} xy \, dy \, dx$.

21. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} \frac{z}{(x+y)^2} dz\, dy\, dx$.

22. State Pappu's second theorem.

Section – III

Answer any **six** questions, each carries 4 mark each.

23. Find Taylor's theorem to find a quadratic approximation of $f(x,y) = \sin x \sin y$ about the origin.

24. The temperature of a point (x,y) on a unit circle is given by $T(x,y) = 1 + xy$. Find the temperature of the two hottest point on the circle.

25. Determine the range of x for which the power series $P(x) = 1 + 2x + 4x^2 + 8x^3 + \dots$ converge.

26. Using Maclaurin series find a power series expansion for $f(x) = \log(1+x)2$.

27. The position vector of a particle at time t is given by $\vec{r}(t) = 2t^2\hat{i} + (3t-2)\hat{j} + (3t^2-1)\hat{k}$. Find (a) Unit tangent vector $\hat{t}$ at $t=1$ (b) Speed at $t=1$ (c) acceleration $\vec{a}$ at $t=1$.

28. Find curl and divergence of $F = x^2y^2\hat{i} + y^2z^2\hat{j} + x^2z^2\hat{k}$ at (1, 1, 1)

29. For the function $\varphi = x^2y + yz$ at the point (1, 2, –1). Find its rate of change with distance in the direction $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$. At this same point what is the greatest possible rate of change with distance and which direction does it occurs.

30. Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$.

31. A tetrahedron is bounded by three coordinate surface and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and has density $\rho(x,y,z) = 1 + \frac{x}{a}$. Find the average density.

Section – IV

Answer any two questions, each carries 15 marks each.

32. (a) Find the greatest and smallest value of $f(x,y) = xy$ which takes the values on ellipse $x^2 + 4y^2 = 8$.

    (b) Find the point closest to the origin on the line of intersection of the planes $y + 2z = 12$ and $x + y = 6$.

33. Determine the convergence of the following series

    (a) $\sum_{n=1}^{\infty} \frac{1}{\left(n - \frac{3}{2}\right)^2}$

    (b) $\sum_{n=1}^{\infty} \frac{1}{n!+1}$

    (c) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

34. (a) Find and expression for a volume element in spherical polar coordinates and hence calculate the moment of inertia about the diameter of a uniform sphere of radius '*a*'.

    (b) Evaluate the double integral $I = \iint_R \left(a + \sqrt{x^2 + y^2}\right) dx\,dy$ where R is the region bounded by the circle $x^2 + y^2 = a^2$.

35. (a) Find the expression for the equation of the tangent plane and normal line to the surface $\varphi(x,y,z) = c$ at the point P with coordinates (x_0, y_0, z_0). Hence find the equation of tangent plane and normal line to the surface of the sphere $x^2 + y^2 + z^2 = a^2$ at $(0, a, 0)$

    (b) Show that $\nabla \cdot (\nabla \phi \times \nabla \varphi) = 0$ where ϕ and φ are scalar fields.
