**S – 3421**

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Reg. No. : ..............................

Name : ..................................

# Third Semester B.Sc. Degree Examination, February 2024

## First Degree Programme under CBCSS

## Mathematics

## Complementary Course for Physics

## MM 1331.1 MATHEMATICS III — LINEAR ALGEBRA, SPECIAL FUNCTIONS AND CALCULUS

**(2021 Admission onwards)**

Time : 3 Hours

Max. Marks : 80

SECTION – I

(All the first **ten** questions are compulsory. They carry **1** mark each).

1. Write the differential equation corresponding to $y = a\cos nx$.

2. State Cayley - Hamilton theorem.

3. Find the rank of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

4. Which one of the following matrices is in the reduced echelon form?

$$\begin{bmatrix} 1 & 2 & 3 & 7 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 5 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

5. Verify that $y = e^{-2x}$ is a solution of $y'' + y' - 2y = 0$.

6. Find the particular integral of $y'' + 9y = e^{2x}$.

7. Define an inverse square field.

8. Determine whether the force field $F = 4y\, i + 4x\, j$ is a conservative or not.

9. If $\sigma$ any closed surface enclosing a volume V and $r = xi + yj + zk$, prove that $\iint_{\sigma} r.n\, dS = 3V$.

10. Find $\beta(1,1)$.

**($10 \times 1 = 10$ Marks)**

SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Show that a square matrix A can be expressed as a sum of two matrices of which one is symmetric and the other is Skew symmetric.

12. Find A and B if $A + B = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

13. If C is the straight line path from $(0, 0, 0)$ to $(1, 1, 1)$, then evaluate $\int_C dx + 2dy + 3dz$.

14. Show that $x = a\cos nt$ is a solution of the differential equation $\frac{d^2 x}{dt^2} + n^2 x = 0$.

15. State Greens theorem including all hypotheses.

16. Solve $\frac{dy}{dx} + y\tan x = \cos x$.

17. Solve $(y'' + y' + 1)^2 = 0$.

18. Find the work done in moving a particle in the force field $F = 3x^2 i + (2xz - y) j - zk$ from $t = 0$ to $t = 1$ along the curve $x = 2t^2$, $y = t, z = 4t^3$.

19. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, find $A^2$.

20. Find the sum and product of eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

21. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, find its characteristic equation of A.

22. Find the outward flux of the vector field $F(x,y,z) = 2x\,i + 3y\,j + z^2\,k$ across the unit cube $x=0,\ y=0, z=0,\ x=1,\ y=1,\ z=1$.

**(8 × 2 = 16 Marks)**

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

23. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 4 & -5 \\ 1 & -2 \end{bmatrix}$.

24. Using Cayley Hamilton theorem evaluate $A^{-1}$ given $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

25. Find the orthogonal trajectories of the family of curves $x^2 - y^2 = c^2$.

26. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

27. Solve $(x+y)\frac{dy}{dx} = y - x$.

28. Find the number of solutions of the following system of equations

$2x + 6y + 11 = 0$

$4x + 14y - z - 7 = 0$

$6y - 3z + 2 = 0$

29. Evaluate by stokes theorem $\oint_C \left(e^x\,dx + 2y\,dy - dz\right)$, where C is the curve $x^2 + y^2 = 4$, $z = 2$.

30. Using Gauss's divergence theorem evaluate $\iint_S F.n\,ds$ for $F = x^2 i + y^2 j + z^2 k$ taken over the region V of the cube bounded by $x=0,\ x=1,\ y=0,\ y=1, z=0, z=1$.

31. Show that $\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

**(6 × 4 = 24 Marks)**

## SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

32. Diagonalize the symmetric matrix $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$.

33. (a) Find for what values of $a$ and $b$, the equations

$$x + 2y + 3z = 6$$
$$3x + 3y + 5z = 9$$
$$2x + 5y + az = b$$

have

(i) no solution

(ii) a unique solution

(iii) more than one solution?

(b) Find the value of k for which the equations

$$2x + 3y + 4z = 0$$
$$x + 2y - 5z = 0$$
$$3x + 5y - kz = 0$$

have a non-trivial solution.

34. Verify Greens theorem for $\int_C (xy + y^2)\,dx + x^2\,dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

35. (a) Solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^x$

(b) Solve $\dfrac{dy}{dx} + \dfrac{x}{1-x^2}\,y = x\sqrt{y}$.

**(2 × 15 = 30 Marks)**