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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Core Course XIII

MM 1645 : INTEGRAL TRANSFORMS

(2018 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find $\mathcal{L}\{te^{2t}\}$.

2. Find $\mathcal{L}\{\sin t \cos t\}$.

3. If $\mathcal{L}\{f(t)\} = F(s)$, then find $\mathcal{L}\{tf(t)\}$.

4. Write the second shifting property of Laplace transform.

5. Find $\mathcal{L}^{-1}\left\{\frac{2+s}{s^2+1}\right\}$.

6. Find the period of $f(x) = \cos 2x$.

7. Write an example for even function.

8. Write the Fourier series expansion of an even periodic function with period 2π.

9. Write the Fourier sine transform of $e^{-ax}, a > 0$.

10. Define Fourier transform of a function $f(x)$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These question carries **2** marks each.

11. Using Definition, find Laplace transform of $f(t) = t$.

12. Find $\mathcal{L}\{\sinh t + 3\cos 2t + te^t\}$.

13. State property : Laplace Transform of derivative of a function. Hence write the Laplace transform of second derivative $F''(t)$ of $f(t)$.

14. Find $\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s-1)}\right\}$.

15. Using the concept of Laplace transform find $\int_{0^+}^{\infty} e^{-2t} t \cos t \, dt$.

16. Using the Laplace transform of integral, find $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+w^2)}\right\}$.

17. Write second shifting theorem of Laplace transform.

18. Find Fourier series of the function $f(x) = x; -\pi < x < \pi$.

19. Write the Fourier series expansion of a 2T periodic function defined in $(-T, T)$.

20. Find Fourier cosine series of $f(x) = e^{2x}; 0 < x < 1$.

21. Find Fourier cosine transform of $f(x) = \begin{cases} k \,;\, 0 < x < a \\ 0 \,;\, \quad\;\; x > a \end{cases}$

22. Show that $\mathcal{F}_C\{f'(x)\} = w\,\mathcal{F}_S\{f(x)\} - \sqrt{\frac{2}{\pi}}\,f(0)$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Define Dirac's Delta function and find its Laplace transform.

24. State and prove first shifting theorem of Laplace Transform.

25. Using Laplace transform, solve the differential equation $y'' + 4y = 4t$, $y(0) = 1$ and $y'(0) = 5$.

26. Using Laplace transform solve the integral equation $y(x) = x^3 + \int_0^x \sin(x-t)\,y(t)\,dt$.

27. Evaluate $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$, using convolution property.

28. Represent function $f(x) = \begin{cases} -1 & for\ -\pi < x < 0 \\ 0 & for\ x = 0 \\ 1 & for\ 0 < x < \pi \end{cases}$ as a Fourier series.

29. Find Fourier integral representation of function of $f(x) = \begin{cases} 1 & if\ |x| < 1 \\ 0 & if\ |x| > 1 \end{cases}$

30. Find the Fourier Transform of $f(x) = 1$ if $|x| < 1$ and $f(x) = 0$ otherwise.

31. Show that : $\mathcal{F}\{f'(x)\} = iw\,\mathcal{F}\{f(x)\}$, where $f(x) \to 0$ as $|x| \to \infty$ and $f'(x)$ is absolutely integrable over x.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These question carries **15** marks each.

32\. Find

(a) $\mathcal{L}\{f(t)\}$ if $f(t) = \begin{cases} 1, 0 < t < \pi \\ 0, \pi < t < 2\pi \\ \sin t,\ t > 2\pi \end{cases}$

(b) $\mathcal{L}\left\{\frac{\sin t}{t}\right\}$

(c) $\mathcal{L}^{-1}\left\{\ln \frac{s+a}{s+b}\right\}$

33\. (a) Deduce a formula to calculate the Laplace transform of the n^{th} derivative $f^{n}(t)$ of a function $f(t)$.

(b) Using Laplace transform solve the system of differential equations

$y_1' + y_2 = 1$ and $y_2' - y_1 + 4e^{t} = 0$ given $y_1(0) = 0$, $y_2(0) = 0$.

34\. Find half range Fourier sine and cosine series of $f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$

35\. Using Fourier integral representation, show that $\int_0^\infty \frac{\cos\frac{\pi\omega}{2}}{1-\omega^2} \cos \omega x \, d\omega = \begin{cases} \frac{\pi}{2}\cos x; & |x| \le \frac{\pi}{2} \\ 0 & ; |x| > \frac{\pi}{2} \end{cases}$.

(2 × 15 = 30 Marks)
