(Pages : 4)

T – 1612

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2024

First Degree Programme under CBCSS

Mathematics

Elective Course

MM 1661.1 : GRAPH THEORY

(2018 Admission Onwards)

Time : 3 Hours | Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory

They carry **1** mark each.

1. What is an isolated vertex?
2. Define a complete graph.
3. State first theorem of graph theory.
4. Define trail.
5. Draw a tree with five vertices.
6. A graph G is called Hamiltonian if ————————
7. State travelling salesman problem.
8. State Euler's formula for plane graphs.
9. Define degree of a face of a plane graph.
10. Give an example of an Eulerian graph.

(10 × 1 = 10 Ma

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SECTION – II

Answer any **eight** questions

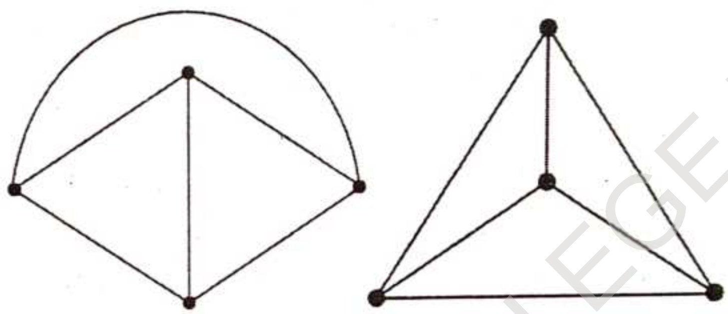
These questions carry **2** marks each.

11. Draw K_4.

12. Prove that in a tree, there is precisely one path between two distinct vertices.

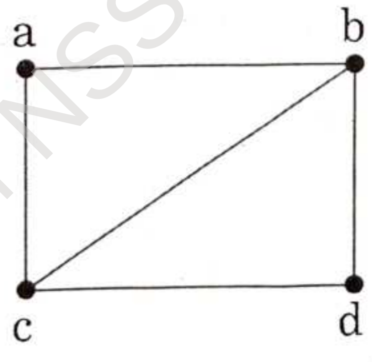
13. Prove that if G is a tree with n vertices, then G is an acyclic graph with $(n-1)$ edges.

14. Show that the following two graphs ate isomorphic:



15. Define k-regular graph and draw a 2-regular graph.

16. Show that in the following graph, sum of degrees of vertices is even.



17. Prove that if a connected graph G is Euler, then the degree of every vertex is even.

18. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.

19. Prove that redrawings of the same planar graph have same number of faces.

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20. Explain travelling salesman problem.

21. Draw a planar graph and show that it's subgraphs are also planar.

22. State Kuratowski's theorem.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions

These questions carry **4** marks each.

23. Define graph isomorphism and give two isomorphic graphs with four vertices.

24. What is a spanning subgraph? Draw a spanning subgraph of K_4.

25. Prove that any tree with at least two vertices has more than one vertex of degree one.

26. Prove that a connected graph is a tree if and only if every edge is a bridge.

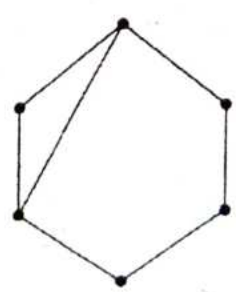
27. Prove that if a simple graph with at least three vertices is 2-connected if for each pair of distinct vertices *u* and *v* of *G*, there are two internally disjoint *u*-*v* paths in *G*.

28. Explain Chinese Postman problem.

29. Draw a Hamiltonian graph with six vertices.

30. Show that $K_{3,3}$ is non-planar.

31. Draw the closure of the following graph:



(6 × 4 = 24 Marks)

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SECTION – IV

Answer any **two** questions

These questions carry **15** marks each.

32. Prove that a graph G is connected if and only if it has a spanning tree.

33. Prove that a tree with n vertices has precisely $(n-1)$ edges.

34. Prove that a connected graph G is Euler if and only if degree of every vertex of G is even.

35. Prove that if G is a connected plane graph and let n, e and f denote the number of vertices, edges and faces of G respectively, then $n-e+f=2$.

(2 × 15 = 30 Marks)

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