V T M N S S COLLEGE DHANUVACHAPURAM

Third Semester B.Sc. Degree Mathematics Question Bank

Complementary Course for Chemistry

1331.2: Linear Algebra, Probability Theory & Numerical Solutions

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2 Mark

1. Show that if A is a square matrix
   (i) A+A' is Symmetric
   (ii) A-A' is Skew Symmetric
2. Find the sum and product of eigen values of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
3. If A and B are matrices such that A+B=$\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and A-B=$\begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$.Find A and B.
4. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$
5. State Cayley-Hamilton theorem and find the characteristic equation of $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$.
6. Find the eigen value of the matrix $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$.
7. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$.
8. Show that for any square matrix A, A and A' have the same eigen values.
9. If A=$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, show that $A^2 - 4A - 5I = 0$.
10. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$
11. Find the sum and product of eigen values of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$
12. Find the rank of the matrix $\begin{bmatrix} 2 & 4 & 6 \\ 4 & 8 & 12 \end{bmatrix}$.
13. Evaluate the determinant $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$.

14. Find the rank of the matix $\begin{bmatrix} 2 & -3 & 5 & 3 \\ 4 & -1 & 1 & 1 \\ 3 & -2 & 3 & 4 \end{bmatrix}$.
15. In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.
16. How many 4 digit numbers can be formed from the 6 digits 2,3,5,6,7,9 without repetition. How many of them are less than 500?
17. Find the probability of exactly 52 heads in 100 tosses of a coin using a binomial distribution.
18. Find the probability that a single card drawn from a shuffled deck of cards will be either a diamond or a king or both.
19. A committee consists of 9 students 2 of which are from the first year, 3 from second year and 4 from third year. 3 students are to be removed at random. What is the change that (i) the 3 students belong to different classes.

(ii) two belong to the same class and third to a different class.

20. Given A= (3i, 1-i, 2+3i, 1+2i), B= (-1, 1+2i, 3-i , i). Find Inner product of A and B.
21. Two students are working separately on the same problem. If the first student has probability $\frac{1}{2}$of solving it and the second student has probability$\frac{3}{4}$ off solving it. What is the probability that at least one of them solves it?
22. What is the probability of getting a king of red colour from a well shuffled deck of 52 cards?
23. What is the chance that a leap year selected at random will contain 53 Sundays?
24. Evaluate $p(A/B)$ and $p(B/A)$ given $p(A) = 1/4$ and $p(B) = 1/3$.
25. In 256 sets of 12 tosses of coin, in how many cases, one can expect 8 heads and 4 tails?
26. Find the number of permutations of all the letters of the word "MATHEMATICS"
27. Use a binomial distribution to calculate *P(X=0)* and *P(X=1)*.
28. Suppose 5 cards are drawn at random from a pack of 52 cards. If all cards are red, find the probability that all of them are hearts.
29. Define mutually exclusive events.
30. There are 10 chairs in a row and 8 people to be seated. In how many ways can this be done?
31. Let A and B be two events with P(A)=12 ,P(B)=13, P(AB)=14 . Find $P(A^1/B^1)$
32. Evaluate $\Delta \tan^{-1} x$
33. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal Rule
34. Find the cubic polynomial which takes the following values.

x	0	1	2	3
f	1	2	1	10

35. If Aand B are independent. $P(A) = 0.6, P(B) = 0.5$, find $P(A^c \cap B^c)$

36. The probability density function of a variable X is

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

Find $(X \geq 5)$.

37. Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places.

38. Evaluate $\sqrt{5}$ by Newton's iteration method.

39. Find the missing term in the table

x	2	3	4	5	6
y	45	49.2	54.1	-	67.4

40. Find a solution using Simpson's 1/3 rule.

x	0	0.1	0.2	0.3	0.4
f(x)	1	0.9975	0.9900	0.9776	0.8604

4 Mark

1. Find the eigen values and eigen vectors of the matrices (i)$\begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}$; $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$
2. Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.
3. Find x,y,z and w given that $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 5 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 6 & x+y \\ z+w & 5 \end{bmatrix}$.
4. Show that the matrix $\begin{bmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & -1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$ is orthogonal.
5. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.
6. Verify Cayley- Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

7. Evaluate the determinant D=$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$

8. Find the eigen values and eigen vectors of the matrix A=$\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

9. Find the characteristic equation of the matrix A=$\begin{bmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{bmatrix}$. Hence find A^{-1}.

10. If A=$\begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$, find A^2 and hence find A^n.

11. Using Cramer's rule solve the set of equations:

$$2x + 3y = 3$$

$$x - 2y = 5$$

12. Write and row reduce the augmented matrix for the equations:

$x - y + 4z = 5$

$2x - 3y + 8z = 4$

$x - 2y + 4z = 9.$

13. Using Cayley Hamilton theorem evaluate A^{-1}, given A=$\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

14. Find the number of solutions of the following system of equations

2x+6y+11=0 , 6x+20y-6z+3=0, 6y-18z+1=0.

15. Fit a Poisson distribution to the set of observations

X:	0	1	2	3	4
F:	122	60	15	2	1

16. (a) A biased coin is tossed till a head appears for the first time. What is the probability that the number of required tosses is odd.

(b) Two persons A and B toss an unbiased coin alternately on the understanding that the first who gets the head wins. If A starts the game, find their respective chances of winning.

17. What is the probability that a number n, $1 \le n \le 99$ is divisible by

(a) Both 6 and 10

(b) By either 6 or 10 or both.

Compute the mean, the variance and the standard deviation for x.

18. A random variable x takes the values 0,1,2,3 with probabilities $\frac{5}{12}, \frac{1}{3}, \frac{1}{12}, \frac{1}{6}$

19. A letter is selected at random with $1 \leq N \leq 100$. What is the probability that N is divisible by 11? That $N > 90$? That $N \leq 3$? That N is a perfect square?
20. Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second queen if the first card is

    (a) replaced (b) Not replaced
21. A club consists of 50 members. In how many ways can a president, vice president, secretary and treasurer be chosen? In how many ways can committee of 4 members be chosen?
22. Three identical boxes contain red and white balls. The first box contains 3 red and 2 white balls, the second box has 4 red and 5 white balls, and the third box has 2 red and 4 white balls. A box is chosen very randomly and ball is draen from it. If the ball is drawn out is red, what will be the probability that the second box is chosen?
23. Two students are working separately on the same problem. If the first student has the probability ½ of solving it and the second student has the probability ¾ of solving it, what is the probability hat atleasst one of them solves it?
24. Which is the mst probable sum in a toss of two dice? What is its probability?
25. A die is tossed thrice. A success is "getting 1 or 6" on a toss. Find the mean and variance of the number of successes.
26. A problem in mathematics is given to 3 students A,B,C, whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem will be solved.
27. A club contains 30 members -20 male and 10 female. In how many ways can a committee of 3 with atleast 1 women be selected?
28. Using Baye's formula find the probability of all heads in thre tosses of a coin if you know that atleast one is head?
29. Using Newton's Iterative method, find the real root of $x \log_{10} x = 1.2$ correct to 4 decimal places.
30. If $y_{10} = 3, y_{11} = 6, y_{12} = 11, y_{13} = 18, y_{14} = 27$, find y_4.
31. Find the cubic polynomial which takes the following values:

x	0	1	2	3
f(x)	1	2	1	10

    Hence evaluate *f(4).*
32. Find the Taylor's series method the value of y at x= 0.1 to 3 places of decimals from

$$\frac{dy}{dx} = x^2y - 1; y(0) = 1$$

33. Use Trapezoidal rule to estimate the integral $\int_0^2 e^{x^2}$dx taking 10 intervals.

34. Find the numerical solution of the equation $\frac{dy}{dx} = 2y^{3/2}$, y(0)=1 for x=0.1 to 0.5 in step of 0.1.

35. Evaluate $I = \int_0^2 (x^2 - 3x + 4)dx$ using trapezium rule with h=0.5.

36. Find y(0.2) for $y' = x^2y - 1$, y(0)=1 with step length 0.1 using Taylor series method.

37. Evaluate the integral $I = \int_0^1 \frac{1}{1+x^2} dx$ using the Simpson's rule.

38. Using Simpson's rule with h=1, evaluate $\int_0^2 \frac{1}{4}\pi x^4 cos\frac{\pi x}{4} dx$.

39. Write a short note on Gass-Seidel iteration.

40. Solve the equation by Gauss Jordan method

$$10x - 7y + 3z + 5u = 6$$
$$-6x + 8y - z - 4u = 5$$
$$3x + y + 4z + 11u = 2$$
$$5x - 9y - 2z + 4u = 7$$

15 Mark

1. (a) Find for what values of a and b, the equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b \text{ have}$$

(i) No solution

(ii) a unique solution

(iii) more than one solution

(b) Find the values of k for which the equations

$3x + y - kz = 0$

$4x - 2y - 3z = 0$

$2kx + 4y + kz = 0$ may possess non-trivial solution.

2. Diagonalize the symmetric matrix A=$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

3. Find the matrix P which transforms the matrix A=$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. Hence calculate A^4.

4. Reduce the matxix A=$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to normal form and hence find the rank.

5. Find the eigen values and eigen vectors of the matrix M=$\begin{bmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{bmatrix}$.

6. Investigate the value of λ and μ so that the equations
   $2x + 3y + 5y = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$ have
   (a) No solution (b) a unique solution (c) an infinite number of solutions.

7. (a) Find the value of a and b for which the equations

$$x + ay + z = 3$$
$$x + 2y + 2z = b$$
$$x + 5y + 3z = 9$$

is (i)consistent and have unique solution

(ii)is inconsistent

(iii)is consistent and have infinitely many solutions

8. Find the value of k for which

$$(3k - 8)x + 3y + 3z = 0$$
$$3x + (3k - 8) + 3z = 0$$
$$3x + 3y + (3k - 8) = 0$$

have a non-trivial solution.

9. (a).The number of particles emitted each minute by a radioactive source is recorded for a period of 10 hours; a total of 1800 counts are registered. During how many 1- minute intervals should we expect to observe no particles; exactly one; etc.?

   (b). If 1500 people each select a number at random between 1 and 500. What is the probability that 2 people selected the number 29?

10. A preliminary test is customarily given to the students at the beginning of a certain course. The following data are accumulated after several years:

   (a) 95% of the students pass the course, 5% fail.

   (b) 96% of the students who pass the course also passed the preliminary test.

(c) 25% of the students who fail the course passed the preliminary test.

What is the probability that a student who has failed the preliminary test will pass the course?

11. If two dice are rolled:

(a) What is the probability that the sum of the numbers on the dice will be 5?

(b) What is the probability that the sum on the dice is divisible by 5?

(c) What is the most probable sum in a toss of two dice? What is its probability?

(d) What is the probability that the sum on the dies is greater than or equal to 9?

(e) If the sum is odd, What is the probability that it is equal to 7?

12. Derive the Poison density function $P_n = \frac{\mu^n}{n!} e^{-\mu}$.

13. (a) There are 3 bags first containing 1 white,2 red,3 green balls; second 2 white,3 red,1 green balls and third 3 white,1 red,2 green balls. Two balls are drawn from a bag chosen at random. These are fixed to be 1 white and 1 red. Find the probability that the balls so drawn came from the second bag.

(b) In a normal distribution, 31% of the items are under 45 and 8% are over 64.Find the mean and standard deviation of the distribution.

14. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
p(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(a) Find the value of k.

(b) Evaluate

i) $P(x < 6)$

ii) $P(x \geq 6)$

iii) $P(0 < x < 5)$

15. Explain :

(i) Stratified sampling method

(ii) Control variates method

(iii) Hit or miss method

16. Using Newton's iterative method, find the real root of the equation $3x = cosx + 1$.

17. Apply Gauss-Jordan method to solve the equations

$x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40.$

18. Use Simpson's 1/3rd Rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking 7 ordinates.

19. Solve the simultaneous equations :

$$x_1 + 6x_2 - 4x_3 = 8, 3x_1 - 20x_2 + x_3 = 12, -x_1 + 3x_2 + 5x_3 = 3$$

Using Gaussian elimination.

20. Use Runge-Kutta method of order 4, find y(0.2) given that $\frac{dy}{dx} = 3x + \frac{y}{2}, y(0) = 1$,taking h = 0.1.
