(Pages : 4) **P – 3817**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme Under CBCSS

Statistics

Complementary Course for Mathematics

ST 1331.1 — PROBABILITY DISTRIBUTIONS AND THEORY OF ESTIMATION

(2014-2017 Admission)

Time : 3 Hours Max. Marks : 80

PART – I

Answer **all** questions. **Each** question carries 1 mark.

1. If X follows the Binomial distribution $B(n, 0.35)$ then find the distribution of $Y = n - X$.

2. If mean of the Poisson distribution is 3, what is the standard deviation?

3. State lack of memory property of exponential distribution.

4. Define standard normal distribution.

5. Define convergence in probability.

6. What is the relation between mean and variance of Chi square distribution?

7. Define standard error.

P.T.O.

8. If X follows a standard normal distribution then what is the distribution of X^2?

9. A normal population has a mean of 75 and a standard deviation of 8. A random sample of 800 is selected. What is the expected value of $\bar{x}$?

10. Define the concept of efficiency.

(10 × 1 = 10 Marks)

PART – II

Answer any **eight** questions. **Each** question carries **2** marks.

11. If X is a random variable with a continuous distribution, find the distribution of Y $F(x)$, where $F(x)$ is the distribution function of X.

12. Find the mode of the distribution $p(x) = \left(\frac{1}{2}\right)^x$, $x = 1,2,3,\ldots$.

13. X is a normal variate with mean 20 and S.D 5. Find $P(16 \leq X \leq 22)$.

14. If X has a uniform distribution over $[0,1]$, show that $Y = -2 \log X$ follows exponential distribution.

15. If $X \sim N(5, 3)$, find the distribution of $Y = 2X + 5$.

16. State Lindberg Levy form of Central limit theorem.

17. What is meant by sampling distribution?

18. Mention any two properties of MLE.

19. State factorization theorem.

20. For the geometric distribution $f(x;\theta) = \theta(1-\theta)^{x-1}$, $x = 1,2,\ldots,0 < \theta < 1$. Obtain an unbiased estimator of $1/\theta$.

21. Let X_1 *and* X_2 be a random sample of size 2 from $N(0, 1)$. Then find the distribution of $\frac{(x_1 + x_2)^2}{(x_1 - x_2)^2}$.

22. If $x_1, x_2, \ldots, x_n$ is a random sample from $N(\mu, 1)$,find sufficient estimator for μ.

(8 × 2 = 16 Marks)

PART – III

Answer any **six** questions. **Each** question carries **4** marks.

23. An unbiased die is thrown. Let X denotes the number thrown. Find the mean and variance of X.

24. State and prove the additive property of gamma distribution.

25. If X following the Beta distribution of first kind with parameters p and q, find the distribution of $Y = \frac{x}{1-x}$.

26. The mean and variance of a binomial variate X are 16 and 8. Find (a) $P(X = 0)$, (b) $P(X \geq 2)$.

27. If X is an exponential distribution with parameter θ, obtain the m.g.f. of X and hence obtain the mean of the distribution.

28. Establish the relation between normal, Chi square, t and F distributions.

29. Define χ^2 distribution with n degrees of freedom. Derive its mean and variance.

30. Explain least square method of estimation.

31. Obtain confidence interval for mean for a random sample of size n from normal population with unknown variance, when the sample size is large.

(6 × 4 = 24 Marks)

PART – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. The number of death reported of people of age more than hundred years on 1000 days were noted.

No. of deaths reported :	0	1	2	3	4	5	6	7	8
No. of days :	229	325	257	119	50	17	2	1	0

Fit a Poisson distribution and calculate the theoretical frequencies.

33. (a) State and prove Chebychev's inequality.

(b) If X follows a binomial distribution with $n = 100$ and $p = 1/2$, obtain using Chebychev's inequality a lower limit for $P\{|X - 50| < 7.5\}$.

34. (a) If $X_1, X_2, \ldots, X_n$ is a random sample from a normal population with mean μ and variance σ^2. Obtain the distribution of sample mean.

(b) Let $X_1, X_2, \ldots, X_n$ is a random sample from a normal population with mean μ. and variance σ^2. Find the moment estimator of μ and σ^2.

35. (a) Explain Cramer Rao lower bound.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with pdf $f(x, \theta) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$.

Find CRLB for the variance of the unbiased estimator of θ.

(2 × 15 = 30 Marks)