VTM NSS COLLEGE DHANUVACHAPURAM

Department Of Mathematics

Question Bank

4th Semester Maths for Chemistry

MM 1431.2 Differential Equations, Vector Calculus and Abstract Algebra

2 Marks Questions

1 Write the differential equation corresponding to $y = a\cos x$

2. The integrating factor of $y' - y = e^{2x}$ is ____

3 The order and degree of the differential equation $\frac{d^2y}{dx^2} - 4\left(\frac{dy}{dx}\right)^3 = 0$ is ____

4 Write the general form of linear differential equation of first order

5 Form the differential equation of the function $x = A\cos(nt + \alpha)$

6 Verify whether the equation $xy\,dx + (2x^2 + 3y^2 - 20)dy = 0$ is exact or not.

7 Solve $y' = \frac{-y}{x}$, given $y(1) = 1$

8 Solve $(y' + y' + 1)^2 = 0$

9 Show that $x = a\cos nt$ is a solution of the differential equation $\frac{d^2x}{dt^2} + n^2x = 0$

10 Define Wronskian.

11 Prove that the force field $F = ie^y + jxe^y$ is conservative in the entire xy plane.

12. Define an inverse square field

13 State Divergence theorem.

14 State Green's theorem

15. State Stoke's theorem

16 If C is the straight line path from $(1,2,3)$ to $(4,5,6)$ then evaluate $\int_C dx + 2dy + 3dz$.

17 Using Green's theorem evaluate $\int_C 4xy\,dx + 2xy\,dy$ where C is the rectangle bounded by $x=-2$, $x=4$, $y=1$, $y=2$

18 Use Divergence theorem to find the outward flux of the vector field $F(x,y,z) = 2xi + 3yj + z^2k$.

19. Find the divergence and curl of the vector field $F(x,y,z) = x^2yi + 2y^3zj + 3zk$

20 Determine whether the vector field $F(x,y,z) = (y+x)i + (y-x)j$ is conservative ?

21 Define conservative vector field and potential function.

22 Define a ring.

23 Express i^{23} in the form $a+ib$

24 Solve $x+5=3$ in Z_8.

25 State and prove associativity of composition of functions f, g, h.

26 Find all solutions in C of the equation $z^2 = i$

27 Draw the subgroup diagram for the Klien 4-group V.

28 Prove that every cyclic group is abelian.

29 Define a binary operation.

30 Define a cyclic group

31 Prove that every cyclic group is abelian.

32 Define subgroup.

33 Write the left and right cancellation laws.

34 Define a group.

35 Compute $(2-5i)(8+3i)$.

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4 Marks Questions.

36 Solve $x\frac{dy}{dx} + y = xy^3$

37 Solve $1 + yx\frac{dx}{dy} + x^2 = 0$ using varIable seperable method

38 Solve $(x^2-y^2)dx - xy\,dy = 0$.

39 Solve $(x+1)\frac{dy}{dx} - ye^{3x}(x+1)^2 = 0$

40 Solve $x\frac{dy}{dx} + y = x^3y^6$

41 Solve $(1+2xy\cos x^2 - 2xy)dx + \sin(x^2 - x^2)dy = 0$

42 Solve i) $(D^4 - 4D + 4)y = 0$, ii) $\frac{d^4x}{dt^4} + 4x = 0$.

43 Find the Particular Integral of $(D+2)(D-1)^2y = e^{-2x} + 2\sinh x$.

44 Solve $(D-2)^2 = 8(e^{2x} + \sin 2x + x^2)$

45 Solve by the method of variation of parameters, $\frac{d^2y}{dx^2} - y = \frac{2}{(1+e^x)}$.

46 Solve $(3y+2x+4)dx - (4x+6y+5)dy = 0$.

47 Solve $\frac{y}{x}\frac{dy}{dx} + \frac{x^2+y^2-1}{2(x^2+y^2)+1} = 0$.

48. Using stoke's theorem evaluate $\int_C (x+y)dx + (2x-z)dy + (y+z)dz$ where C is the boundary of the triangle with vertices $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$

49 Evaluate $\int_C F\cdot n \, ds$ where $F = 4xi - 2y^2j + z^2k$ and σ is the closed surface consisting of the cylinder $x^2+y^2=4$ and the planes $z=0$ and $z=3$.

50. Verify Green's theorem for $\int_C (xy+y^2)dx + x^2dy$, where C is closed the curve consisting of line $y=x$ and parabola $y=x^2$.

51 Let $F(x,y) = 2xy^3 i + (1+3x^2y^2) j$. Show that F is a conservative vector field on the entire xy-plane. Also find ϕ.

52. Let $F(x,y) = e^y i + xe^y j$. Verify F is conservative. Find the workdone by the field on a particular that moves from $(1,0)$ to $(-1,0)$ along the semicircular path C.

53 Evaluate the surface integral $\iint_\sigma x^2\, dS$ over the sphere $x^2+y^2+z^2=1$.

54 Evaluate the surface integral $\iint_\sigma xz\, dS$ where σ is the part of the plane $x+y+z=1$ that lies in the first octant.

55 Evaluate the surface integral $\iint_\sigma y^2z^2\, dS$ where σ is the part of the cone $z=\sqrt{x^2+y^2}$ that lies between the planes $z=1$ and $z=2$.

56 Find the outward flux of the vector field $F(x,y,z) = x^3 i + y^3 j + z^3 k$ across the surface of the region that is enclosed by the hemisphere $z=\sqrt{a^2-x^2-y^2}$ and the plane $z=0$.

57. Prove that the identity and inverse are unique in a group.

58 Write an example of a group of 4 elements and draw its group table.

59 In a ring R, with additive identity 0, then for any $a, b \in R$, Prove that i) $0.a = a.0 = 0$
ii) $a(-b) = (-a)b = -(ab)$ (iii) $(-a)(-b) = ab$

60 If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$

Find $\tau^2\sigma$

61. Find all solutions of $z^4 = -16$

62 Show that the set R^+ is an abelian group.

63 Show that the set $M_n(R)$ of ~~all nxn~~ consisting of all invertible $n \times n$ matrices under matrix multiplication is a group.

64. Show that the ~~group~~ binary operation $*$ defined on Q^+ by $a*b = \frac{ab}{2}$ is a group.

65. State and prove the left and right cancellation laws in a group G.

~~66 Prove that a subgroup of a cyclic group is cyclic.~~

66 Draw the multiplication table for the symmetric group S_3.

67 Prove that $(Z, +, \cdot)$ is a ring.

68 Find the work performed by the force field $F(x,y,z) = x^2 i + 4xy^3 j + y^2 x k$ on a particle that traverses the rectangle C in the plane $z = y$.

69 Using Divergence theorem, find the outward flux of the vector field $F(x,y,z) = x^3 i + y^3 j + z^2 k$ across the surface of the region that is enclosed by the circular cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$.

70 Using Green's theorem, evaluate the integral $\oint_C (x^2 - 2y^2)\,dx + x\,dy$, where C is the circle $x^2+y^2 = 9$.

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15 Marks

71 a) Solve $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = e^x$

b) Solve $(y-x)\dfrac{dy}{dx} + 2x + 3y = 0$.

72 a) Find the orthogonal trajectories of the family of coaxial circles $x^2+y^2+2\lambda x + c = 0$.

b) Solve : $x^2y'' + 7xy' + 9y = 0$.

73 a) Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$

b) Solve $(D^2 - 1)y = x \sin 3x + \cos x$

74 a) Solve $(p^2 + 2py \cot x) = y^2$

b) Solve $y = 2px + y^2p^3$

75 a) Solve $\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}} - \dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy} = 1$

b) Solve $3x(1-x^2)y^2\dfrac{dy}{dx} + (2x^2-1)y^3 = ax^3$

76 Verify stoke's theorem for the vector field $F(x,y,z) = 2zi + 3xj + 5yk$, taking the surface σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and C to be the positively oriented circle $x^2+y^2 = 4$ that forms the boundary of σ in the xy plane

77 a) Show that the vector field $F = (xy^2+z)i + (x^2y+2)j + xk$ is conservative and find ϕ such that $F = \nabla\phi$.

b) Find the flux of the vector field $F(x,y,z) = xi+yj+zk$ across σ, where σ is the portion of the surface $z = 1-x^2-y^2$ that lies above the xy plane, and suppose that σ is oriented up.

78. a) Use Green's theorem to evaluate the integral $\oint_C (x^2-3y)\,dx + 3x\,dy$, where C is the circle $x^2+y^2=4$.

b) Use a line integral to find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

79. a) Evaluate the line integral $\int_C (xy+z^3)\,ds$ from $(1,0,0)$ to $(-1,0,\pi)$ along the helix C that is represented by the parametric equations $x=\cos t$, $y=\sin t$, $z=t$ $(0\le t \le \pi)$

b) Evaluate $\int_C 2xy\,dx + (x^2+y^2)\,dy$ along the circular arc C given by $x=\cos t$, $y=\sin t$ $(0\le t\le \pi/2)$.

80. a) Suppose that a semicircular wire has the equation $y=\sqrt{25-x^2}$ and that its mass density is $\delta(x,y) = 15-y$. Find the mass of the wire.

b) Find the area of the surface extending upward from the circle $x^2+y^2=1$ in the xy-plane to the parabolic cylinder $z=1-x^2$.

81 Explain the Dihedral Group of a square D_4 and i) Write the elements of D_4 as permutations
ii) Construct the group table for D_4
iii) Find all subgroups of D_4.

82. Show that the area of a region R enclosed by a simple closed curve C is given by

$$A = \frac{1}{2} \oint_C (x\,dy - y\,dx) = \oint_C x\,dy = -\oint_C y\,dx.$$

Hence calculate the area of the ellipse $x = a\cos\phi$, $y = b\sin\phi$.

83a) Consider the permutations σ and μ in S_6:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix} ; \quad \mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$$

Compute $\sigma^2\mu$.

~~84~~b) Show that the fourth roots of unity is a group.

84) a) Construct the cayley table for $U(12)$.
b) Write all subgroups of V, Klein 4-group.

85 a) Find all the generators of Z_{18} and draw the subgroup diagram.
b) Prove that Z^+ with usual addition and multiplication is a ring.