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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XI

MM 1643 : ABSTRACT ALGEBRA-RING THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Give an example of commutative ring with zero divisors.

2. Which are units of Z_5 ?

3. What is the characteristic of nZ ?

4. Find the number of elements in the factor ring $\frac{2z}{8z}$.

5. State factor theorem.

6. Is the ring $2z$ isomorphic to the ring $4z$?

7. Define a primitive polynomial.

P.T.O.

8. Define a unique factorisation domain.

9. Find the norm of $1+\sqrt{-5}$.

10. State whether true or false: Every Euclidean domain is a UFD.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each carries **2** marks.

11. Prove that $a(-b)=(-a)b=-(ab)$ in a ring R.

12. Show that if a and b are idempotents in a commutative ring, then ab is also idempotent.

13. Prove that the ideal $<x^2+1>$ is not prime in $z_2[x]$.

14. Define a principal ideal domain. Give an example.

15. Find the kernel of the ring homomorphism from $R[x]$ to R defined by $f(x) \to f(1)$.

16. Is the field of real numbers is ring isomorphic to the field of complex numbers? Justify your answer.

17. Show that the polynomial $2x+1$ in $Z_4[x]$ has a multiplicative inverse in $Z_4[x]$.

18. Construct a field of 9 elements.

19. Prove that every subring of Z is of the form nZ for some $n \in Z$.

20. Define associates and irreducibles in an integral domain.

21. Suppose that a and b belong to an integral domain, $b \neq 0$ and a is not a unit. Show that $<ab>$ is a proper subset of $<b>$.

22. Let $d < -1$ be an integer, that is not divisible by the square of a prime. Prove that the only units of $Z(\sqrt{d})$ are ± 1.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Prove that the set $\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \mid a, b \in Z\right\}$ is a subring of the ring of all 2 × 2 matrices over Z.

24. Find all solutions of the equation $x^2 - 5x + 6 = 0$ in Z_{12}.

25. Show that an intersection of subfields of a field F is again a sub field of F.

26. Let φ be a ring homomorphism from a ring R to a ring S. If B is an ideal in S, then prove that $\phi^{-1}(B) = \{r \in R \mid \phi(r) \in B\}$ is an ideal of R.

27. Let R be a ring with unity 1. Then show that the mapping $\phi : Z \to R$ by $\phi(m) = m.1$ is a ring homomorphism.

28. Show that if D is an integral domain, then $D[x]$ is also an integral domain.

29. Show that $1 + \sqrt{-3}$ is irreducible in $Z\left[\sqrt{-3}\right]$.

30. Show that the ring of Gaussian integers $Z[i] = \{a + bi \mid a, b \in z\}$ is a Euclidean domain.

31. In $Z\left[\sqrt{-5}\right]$, show that 21 doesn't factor uniquely as a product of irreducibles.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** carries **15** marks.

32. (a) Prove that a finite integral domain is a field.

    (b) Prove that the characteristic of an integral domain is zero or prime.

33. Let D be an integral domain. Then prove that there exist a field F that contains a subring isomorphic to D.

34. (a) State and prove Gauss's lemma.

(b) Let $f(x) \in Z[x]$. Prove that if $f(x)$ is reducible over Q, then it is reducible over Z.

35. Prove that in a PID, an element is irreducible if and only if it is prime.

(2 × 15 = 30 Marks)