

Reg. No. : .....

Name : .....

**First Semester B.Sc. Degree Examination, March 2023**

**First Degree Programme under CBCSS**

**Mathematics**

**Core Course**

**MM 1141 : METHODS OF MATHEMATICS**

**(2018 Admission Onwards)**

Time : 3 Hours

Max. Marks : 80

**SECTION – I**

All questions are compulsory. Each question carries 1 mark.

1. What is the local linear approximation of  $f(x) = \sqrt{x}$  at  $x_0 = 1$ .
2. Define point of inflection.
3. Define critical point.
4. State Extreme value theorem.
5. For a particle in rectilinear motion, the acceleration and position functions  $a(t)$  and  $s(t)$  are related by the equation \_\_\_\_\_
6. Let  $A(x)$  be the area under the graph of a nonnegative continuous function  $f$  over an interval  $[a, x]$ , then  $A'(x) =$  \_\_\_\_\_.
7. Integrals over infinite intervals are known as \_\_\_\_\_

8.  $\cosh x + \sinh x = \text{—————}$ .

9. Define the work done by a force  $F$ .

10. The total mass of a homogeneous lamina of area  $A$  and density  $\delta$  is —————.

(10 × 1 = 10 Marks)

### SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Evaluate  $\lim_{x \rightarrow 0^+} \left( \frac{1}{x} - \frac{1}{\sin x} \right)$ .

12. Find the subintervals of  $[0, 2\pi]$  in which the function  $f(x) = x + 2\sin x$  is decreasing.

13. Find all critical points of  $f(x) = x^3 - 3x + 1$ .

14. What are the geometrical implications of the multiplicity of a root of a polynomial?

15. Find the horizontal and vertical asymptotes of the curve given by  $y = \frac{\ln x}{x}$ .

16. Find the absolute extrema of  $f(x) = 6x^{4/3} - 3x^{1/3}$  on the interval  $[-1, 1]$ .

17. Suppose that a particle moves on a coordinate line so that its velocity at time  $t$  is  $v(t) = t^2 - 2t$  m/s. Find the distance traveled by the particle during the time interval  $0 \leq t \leq 3$ .

18. Find the average value of the function  $f(x) = \sqrt{x}$  over the interval  $[1, 4]$ .

19. Define hyperbolic sine and draw its graph.

20. Define improper integral. Is  $\int_0^3 \frac{dx}{x^2 - 3x + 2}$  an improper integral? Explain.

21. Use Pappus Theorem to find the volume  $V$  of the torus generated by revolving a circular region of radius  $b$  about a line at a distance  $a$  (greater than  $b$ ) from the center of the circle.

22. Evaluate  $\int_0^{\infty} e^{-x} dx$ .

(8 × 2 = 16 Marks)

### SECTION – III

Answer **any six** questions. Each question carries **4** marks.

23. Evaluate  $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$ .

24. Find all the inflection points of  $f(x) = xe^{-x}$ .

25. Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

26. State and prove Rolle's theorem.

27. Find the volume of the solid generated when the region between the graphs of the equations  $f(x) = \frac{1}{2} + x^2$  and  $g(x) = x$  over the interval  $[0, 2]$  is revolved about the  $x$ -axis.

28. Using the notion of surface of revolution, show that the area of the surface of a sphere of radius  $r$  is  $4\pi r^2$ .

29. Find the length of the arc of the curve  $y^2 = x^3$  from the origin to the point  $(1, 1)$ .

30. A spring exerts a force of 5 N when stretched 1 m beyond its natural length.

(a) Find the spring constant  $k$ .

(b) How much work is required to stretch the spring 1.8 m beyond its natural length?

31. Evaluate  $\int_0^{\infty} (1-x)e^{-x} dx$ .

(6 × 4 = 24 Marks)

SECTION – IV

Answer any two questions. Each question carries 15 marks.

32. (a) Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches. 5
- (b) Using Roll's theorem show that between any two real root of  $e^{-x} = \sin x$ , there is at least one real root of  $e^{-x} = -\cos x$ . 5
- (c) Find the points of inflection of the cubic  $y = \frac{a^2 x}{x^2 + a^2}$ . 5
33. (a) Explain the 7 steps in sketching the graph of a rational function. 6
- (b) Sketch the graph of  $y = \frac{x^2 - 1}{x^3}$ . 9
34. (a) Find the length of the curve  $y = \log \sec x$  between the points given by  $x = 0$  and  $x = \pi/3$ . 5
- (b) Find the volume when the loop of the curve  $y^2 = x(2x - 1)^2$  revolves about the x-axis. 5
- (c) Find the area of the surface that is generated by revolving the portion of the curve  $y = x^2$  between  $x = 1$  and  $x = 2$  about the y-axis. 5
35. (a) A space probe of mass  $m = 5.00 \times 10^4$  kg travels in deep space subjected only to the force its own engine. Starting at a time when the speed of the probe is  $v = 1.10 \times 10^4$  m/s. the engine is fired continuously over a distance of  $2.50 \times 10^6$  m with a constant force of  $400 \times 10^5$  N in the direction of motion. What is the final speed of the probe? 6
- (b) Evaluate  $\int_1^4 \frac{dx}{(x-2)^{2/3}}$ . 5
- (c) Find the mass and center of gravity of the lamina bounded by the x-axis, the line  $x = 1$ , and the curve  $y = \sqrt{x}$ . Given  $\delta = 2$ . 4

(2 × 15 = 30 Marks)