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Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XIII

MM 1645 : INTEGRAL TRANSFORMS

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

PART – A

All the **first ten** questions are compulsory. They carry 1 mark each.

1. Find the Laplace transform of $f(t)=e^{-3t}$.

2. If $L[f(t)]=F(s)$, then $L\left\{\int_0^t f(u)\,du\right\}=$ ————————

3. Find the inverse Laplace transform of $\frac{1}{s^2+4}$.

4. Define unit step function.

5. Write $L\{f''(t)\}$ in terms of $L(f)$, $f(0)$ and $f'(0)$.

6. Give an example of a periodic function which is neither odd nor even.

P.T.O.

7. Define Fourier cosine transform of a function $f(t)$.

8. What is the standard form of Fourier series for an even function?

9. Find the fundamental period of the function $\cos(n x)$.

10. If $f(x)$ has period p then find the period of $f(2x)$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Find the Laplace transform of $f(t)=t\sin 2t$.

12. Find $L[e^{-3t}\sin 3t]$.

13. Evaluate $L^{-1}\left[\frac{3}{(s+2)^4}\right]$.

14. Find $L^{-1}\left(\frac{1}{(s+2)(s+3)}\right)$.

15. Solve the differential equation $y''-y=t$, $y(0)=1$, $y'(0)=1$.

16. Define convolution of two functions and find the convolution of 1 and -1.

17. Is $L[f(t)+g(t)]=L(f(t))+L[g(t)]$? Explain.

18. Find the Fourier series of $f(x)=x$ for $0<x<2\pi$.

19. Find the Fourier sine series for the function $f(x)=\pi-x$ in $0<x<\pi$.

20. Find the Fourier cosine transform of the function $f(x)=\begin{cases}k, & 0<x<\pi\\ 0, & x>\pi\end{cases}$.

21. Derive the Fourier transform of $f'(x)$, the derivative of $f(x)$.

22. Check whether the following functions are odd or even

(a) $\dfrac{x}{x^2+1}$

(b) $x^3 \tan \pi x$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

23. Find the Laplace transform of the function $f(t)=\begin{cases} t & t \geq 3 \\ 0 & t<3 \end{cases}$.

24. Find the inverse transform of $(3s+1)/(s-1)(s^2+1)$.

25. Solve the equation $y''+3y'+2y=\delta(t-1),\ y(0)=0, y'(0)=0$.

26. Find the Laplace transform of $e^{-4t}\int_0^t t \sin 3t\, dt$.

27. Find the Laplace transform of $t^2 e^{-3t} \sin 2t$.

28. Find the inverse Laplace transform of $\dfrac{2(e^{-s}-e^{-3s})}{s^2-4}$.

29. Find the inverse of $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$.

30. Find the Fourier series for $f(x)=|x|$ from $x=-\pi<x<\pi$ and deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}.$$

31. Using Fourier sine integral, Show that

$$\int_0^\infty \frac{1-\cos \pi w}{w}\sin x w\, dw=\begin{cases}\frac{\pi}{2} & \text{if } 0<x<\pi\\ 0 & \text{if } x>\pi\end{cases}$$

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks.

32. (a) State and prove convolution theorem of Laplace transforms

(b) Use convolution theorem to find the inverse Laplace transform of $\frac{1}{s^2(s-a)}$.

33. Find the Fourier series of the periodic function $f(x)$ of period 2, where $f(x)=\pi x$ in $0\le x\le 2$ and deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots=\frac{\pi}{4}$.

34. (a) Represent $f(x)=\begin{cases}\sin x, & 0<x<\pi\\ 0, & x>\pi\end{cases}$ as Fourier cosine integral

(b) Find the Fourier integral of $f(x)=\begin{cases}\pi-x & \text{if } 0<x<\pi\\ 0, & \text{otherwise}\end{cases}$

35. (a) Find the Fourier transform of $f(x)=e^{-\frac{x^2}{2}}$. What is your inference?

(b) Find the Fourier transform of $f(x)=\begin{cases}|x| & \text{if } -1<x<1\\ 0, & \text{otherwise}\end{cases}$

(2 × 15 = 30 Marks)