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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XII

MM 1644 : ABSTRACT ALGEBRA II

(2014-2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. Each carries **1** mark.

1. Define normal subgroup.

2. If $\varphi : z \to z_7$ is a homomorphism such that $\varphi(1) = 4$. Find $Ker(\varphi)$.

3. Find the order of the factor group $(z_4 \times z_{12}) / <2> \times <2>$.

4. Define an automorphism.

5. State whether true or false: z/nz is cyclic of order n.

6. Compute $(20)(-8)$ in z_{26}.

7. Describe all units in the ring $z \times z$.

8. Find the characteristic of the ring $3z$.

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9. Calculate $\varphi(1001)$.

10. Find all ideals of z.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each carries **2** marks.

11. Show that $\varphi : s_n \to z_2$ defined by $\varphi(\sigma) = \begin{cases} 0 & \text{if } \sigma \text{ is an even permutation} \\ 1 & \text{if } \sigma \text{ is an odd permutation} \end{cases}$

12. If $\varphi : G \to G'$ is a group homomorphism, prove that $\ker(\varphi)$ is a normal subgroup of G.

13. Prove that every subgroup of an abelian group is normal.

14. Find all subgroups of s_3 that are conjugate to $\{i_0, \mu_2\}$.

15. Compute the factor group $(z_4 \times z_8)/ < (0, 2) >$.

16. If R is a ring with additive identity 0, then prove that $(-a)(-b) = ab$, for all $a, b \in R$.

17. Find a zero divisor of the matrix ring $M_2(z_2)$.

18. Determine whether $z \times z$ with addition and multiplication by components is a field.

19. Show that the fields R and C are not isomorphic.

20. Show that $2^{11, 213} - 1$ is not divisible by 11.

21. Use Fermat's theorem to find the remainder of 37^{49}, when it is divided by 7.

22. Let R be a commutative ring with unity of characteristic 4. Compute and simplify $(a+b)^4$ for $a, b \in R$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** carries **4** marks.

23. Prove that a group homomorphism is a one-to-one map if and only if $\ker(\varphi) = \{0\}$.

24. Show that A_n is a normal subgroup of s_n and compute s_n / A_n.

25. Prove that factor group of a cyclic group is cyclic.

26. Prove that in the ring z_n, the devisors of 0 are precisely those nonzero elements that are not relatively prime to n.

27. Solve the equation $x^2 - 5x + 6 = 0$ in z_{12}.

28. Show that $a^2 - b^2 = (a+b)(a-b)$ for all a, b in a ring R if and only if R is commutative.

29. State and prove fundamental homomorphism theorem for rings.

30. Let R be a commutative ring let $a \in R$. Show that $I_a = \{x \in R / ax = 0\}$ is an ideal of R.

31. Let m be a positive integer and let $a \in z_n$ be relatively prime to m. Prove that for each $b \in z_m$, the equation $ax = b$ har a unique solution in z_m.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each carries **15** marks.

32. Let φ be a homomorphism of a group G into a group G'. Prove the following.

    (a) If $a \in G$, then $\varphi(a^{-1}) = [\varphi(a)]^{-1}$

    (b) If H is a subgroup of G, then $\varphi[H]$ is a subgroup of G'.

    (c) If K' is a subgroup of G', then $\varphi^{-1}(K')$ is a subgroup of G.

33. Let H be a subgroup of G. Prove that the left coset multiplication is well defined by the equation $(aH)(bH) = (ab)H$ if and only if H is a normal subgroup of G.

34. (a) Prove that every field F is an integral domain.

(b) Show that an intersection of subfields of a field F is again a subfield of F.

35. (a) Prove that if a is an integer relatively prime to n, then $a^{\varphi(n)} - 1$ is divisible by n.

(b) Find all solutions of the congruence $zx \equiv 6 \pmod 4$.

(2 × 15 = 30 Marks)