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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XII

MM 1644 : LINEAR ALGEBRA

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries 1 mark.

1. Write the given system of equations in column form

   $5x + 20y = 80$

   $-x + 4y = -64$

2. Find two points on the line of intersection of the three planes $t = 0, z = 0$ and $x + y + z + t = 1$ in four dimensional space.

3. Define a symmetric matrix.

4. Define subspace of a vector space.

5. The columns of a matrix A and n vectors from R^m. If they are linearly independent, what is the rank of A?

6. Write the reflexion matrix that transforms (x, y) to (y, x).

7. If a 3 by 3 matrix has $\det A = \frac{-1}{2}$, find $\det(A^{-1})$.

P.T.O.

8. State whether true or false: The determinant of $S^{-1}AS$ equals the determinant of A.

9. If $\lambda \neq 0$ is an eigen value of A and x is its eigen vector, then show that x is also an eigen vector of A^{-1}.

10. Define a transition matrix.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Write down the upper triangular system of equations. Also find the pivots

$$2x + 3y = 1$$
$$10x + 9y = 11$$

12. Give 2 by 2 matrices A and B such that AB = 0, although no entries of A or B are zero.

13. Describe the null space of $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$.

14. Describe the subspace of $\mathbb{R}^3$ spanned by the two vectors (1, 1, –1) and (–1, –1,1).

15. Check whether the transformation $T : R^3 \to R^3$ defined by $T(V_1, V_2, V_3) = (V_2, V_3, 0)$ is linear.

16. Using the big formula, compute det A from 6 terms. Are rows independent?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

17. The corners of a triangle are (2,1), (3,4) and (0,5). What is the area?

18. Suppose the permutation P takes (1,2,3,4,5) to (5,4,1,2,3). What does P^{-1} do to (1,2,3,4,5).

19. Show that the eigen value of A equals the eigen value of A^T.

20. Factor $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ into $S \Lambda S^{-1}$.

21. Compute A^3 for the Fibonacci matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

22. Prove that two eigen vectors of a Hermitian matrix, if they come from different eigen values one orthogonal to one another.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. For which numbers 'a' does the elimination break down

(i) Permanently (ii) Temporarily $\begin{matrix} ax + 3y = -3 \\ 4x + 6y = 6 \end{matrix}$

Solve for x and y after fixing the second break down by a row exchange.

24. Find all matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that satisfy $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} A$.

25. Reduce to echelon form $\begin{bmatrix} 0 & 0 & 2 \\ 1 & -1 & 1 \\ -1 & 1 & -4 \end{bmatrix}$.

26. Find the range and kernel of $T(V_1, V_2) = (V_1 V_1 + V_2)$.

27. Find x, y and z by Cramer's rule :

$2x + y = 1$

$x + 2y + z = 70$

$y + 2z = 0$

28. Define orthogonal matrix. Prove that if A is an orthogonal matrix, then $\det A = \pm 1$.

29. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$.

30. If $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find A^{100} by diagonalising it.

31. Compute $A^H A$ and AA^H for $A = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Compute the symmetric LDL^T factorisation of $A=\begin{bmatrix}1 & 3 & 5\\ 3 & 12 & 18\\ 5 & 18 & 30\end{bmatrix}$.

(b) Use the Gauss-Jordan method to invert $\begin{bmatrix}1 & 2 & 1\\ 3 & -2 & -4\\ 2 & 3 & -1\end{bmatrix}$.

33. (a) Find the value of α that makes the system solvable and find the solution

$x+y+z=1$

$2x-y+2z=1$

$x+2y+z=\alpha$

(b) Find the dimensions of the column space and raw space of $A=\begin{bmatrix}1 & 1 & 0\\ 0 & 2 & 1\\ 0 & 0 & 0\end{bmatrix}$.

34. (a) Find the dimension and a basis for the four fundamental subspaces for $u=\begin{bmatrix}1 & 2 & 0 & 1\\ 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$

(b) Find the 4 by 4 matrix A that represents a right shift : (x_1, x_2, x_3) is transformed to $(0, x_1, x_2, x_3)$. Also find the left shift matrix B from R^4 back to R^3, transforming (x_1, x_2, x_3, x_4) to (x_2, x_3, x_4).

35. (a) Test the cayley-Hamilton theorem on $A=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$.

(b) Find a diagonal matrix M so that $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ is similar to $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$.

(2 × 15 = 30 Marks)