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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course XI

MM 1643 : COMPLEX ANALYSIS II

(2014 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the **ten** questions are compulsory

1. Classify the singularity of exp $(1/z)$ at $z = 0$.

2. Find a power series representation for $1/z^2$ near $z = 3$.

3. If *f* is analytic at *z*, then show that *f* is infinitely differentiable at *z*.

4. Describe the nature of singularity of $\frac{\sin(z - z_0)}{z - z_0}$ at $z = z_0$.

5. Find the residue of $f(z) = taz\ at\ z = \pi/2$.

6. If z_0 is a pole of a function *f* then find $\lim_{z \to z_0} f(z)$.

P.T.O.

7. Find $\int\limits_{|z|=2} \frac{3}{z-3} dz$.

8. Name the type of singularity of cosec z at $z = 0$.

9. Find the singular points of the function $\frac{z+1}{z^2(z-i)}$.

10. Find $P.V \int\limits_{-\infty}^{\infty} x dx$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions.

11. If $f(z)$ is analytic in $D(\alpha; r)$, and a $\in D(\alpha; r)$ show that there exist functions F and G, analytic in D and such that $F'(z) = f(z)$, $G'(z) = (f(z) - f(a))/(z-a)$.

12. If f is entire and if $f(z) \to \infty$ as $z \to \infty$, then show that f is a polynomial.

13. What kind of singularities have $1/(1-e^z)$ at $z = 2\pi i$.

14. Show that $\oint \frac{dz}{z \sin z} = 0$ where the integration is over C, unit circle about the origin.

15. Find the radius of convergence of $\sum_1^\infty \frac{n! z^n}{n^n}$.

16. Evaluate $\oint\limits_{|z|=\pi} \frac{dz}{z-3i} dz$.

17. Find $\int\limits_{|z|=1} \frac{dz}{z^2+4}$.

18. Show that $\int_{|z|=2} \tan z dz = -4\pi i$

19. Find the residues of $\frac{z+1}{z^2(z-2)}$ at its poles.

20. State Jordan's Lemma.

21. Determine the order m of each pole, and find the corresponding residue B for $f(z) = \left(\frac{z}{2z+1}\right)^3$.

22. Show that $\int_{|z|=1} \exp\left(\frac{1}{z^2}\right) dz = 0$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions.

23. Show that $\operatorname{Re}_{z=\pi i} s \frac{z - \sinh z}{z^2 \sinh z} = \frac{i}{\pi}$.

24. Examine the convergence of $\sum_0^\infty \frac{z^n}{n^2}$.

25. If R is the radius of convergence of $\sum a_n z^n$, what is the radius of convergence of $\sum a_n z^{2n}$?

26. State and prove Cauchy Residue theorem.

27. Evaluate $\oint z^2 \sin\left(\frac{1}{2}\right)$ over the positively oriented circle $|z| = 1$.

28. Find the Cauchy principal value of the integral $\int_{-\infty}^{\infty} \frac{x \sin x dx}{x^2 + 2x + 2}$.

29. Find the singularities and residues of the function $f(z) = (z+1)/(z^2+9)$.

30. Using residues, evaluate $\oint \frac{dz}{z(z-2)^2}$, over the unit circle $|z-2| = 1$.

31. Find $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

32. (a) State and prove Cauchy Integral formula.

(b) Using this, evaluate $\int_{|z|=3} \frac{z^2}{z-2}, \oint_{|z|=\frac{1}{4}} \frac{dz}{(4z^2+1)}$.

33. (a) State and prove mean value theorem.

(b) State and prove maximum modulus theorem.

34. Evaluate $\int_0^{\infty} \frac{x^2}{x^6+1} dx$.

35. Explain the three types of isolated singular points of a complex function with examples. Verify the examples with their series representations.

(2 × 15 = 30 Marks)