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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course – X

MM 1642 : COMPLEX ANALYSIS – II

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define convergence of series of complex numbers.

2. Define Uniform Convergence of a sequence of functions.

3. Define a power series.

4. Define a simple pole.

5. Define residue of $f(z)$ at z_0.

6. Define Improper integral over $[0,\infty]$ of a continuous non negative function $f(x)$.

7. Define the Cauchy principal value of f over $(-\infty,\infty)$

8. What do you mean by local property of a mapping.

P.T.O.

9. Define open mapping property.

10. Define the mapping: Translation

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Prove that $\sum_{j=1}^{\infty} C^j$ converges to $\frac{1}{1-c}$ for $|c| < 1$.

12. Find the Maclaurin series for $\sin z$.

13. Define Cauchy Product of two Taylor series.

14. State M test for uniform convergence.

15. State the necessary and sufficient condition for an analytic function to have a zero of order m at z_0.

16. Define the radius on convergence of a power series.

17. If $f(z) = \frac{P(z)}{Q(z)}$, where $P(z)$ and $Q(z)$ are analytical at z_0 and Q has a simple pole at z_0, while $P(z_0) \neq 0$, derive the formula for $\text{Res}(f; z_0)$

18. Find the residue at $z = 0$ of $f(z) = z^2 \sin\left(\frac{1}{z}\right)$ using Laurent series.

19. Evaluate $p.v. \int_{-\infty}^{\infty} x\, dx$.

20. Show that e^z is locally one to one but not globally.

21. State the Riemann Mapping Theorem.

22. Define the mappings: Rotation and Magnification.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. State the comparison test and show that $\sum_{j=1}^{\infty} \frac{3+2i}{(j+1)^j}$ converges.

24. Find the Taylor series of $Log\, z$ around $z = 1$.

25. Prove that the uniform limit of a sequence of continuous functions defined on a simply connected domain is also continuous.

26. Classify the zeros and singularities of $\sin\left(1 - \frac{1}{z}\right)$.

27. Find the residues at each singularity of $f(z) = \cot z$.

28. Prove that, if $f(z)$ has a pole of order m at z_0, then

$$\operatorname{Res}(f : z_0) = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [(z - z_0)^m f(z)].$$

29. Suppose that $f(t)$ and $M(t)$ are continuous function defined on $[a,b]$, with f complex and M real valued. Prove that if $|f(t)| \le M(t)$ on this interval, then $\left|\int_a^b f(t)\,dt\right| \le \int_a^b M(t)\,dt$.

30. Prove that if $f(z)$ is analytic at z_0, and $f'(z_0) \ne 0$ then $f(z)$ is conformal at z_0.

31. Define a Mobius transformation and show that the inverse of a Mobius transformation is another Mobius Transformation

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. Prove that if $f(z)$ is analytic in the disk $|z - z_0| < R$, then there exists a Taylor series which converges to $f(z)$ for all z in this disk.

33. (a) Expand $e^{(1/z)}$ as a Laurent series about $z = 0$.

(b) Find the Laurent Series for $f(z) = \dfrac{z^2 - 2z + 3}{z - 2}$ in the region $|z - 1| > 1$.

34. State and prove Cauchy Residue Theorem. Using Cauchy Residue Theorem evaluate the integral of $f(z) = \dfrac{1 - 2z}{z(z-1)(z-2)}$ over the circle $|z| = 2$.

35. Evaluate $\displaystyle\int_0^{2\pi} \frac{\sin^2\theta}{5 + 4\cos\theta} d\theta$.

(2 × 15 = 30 Marks)