VTM NSS COLLEGE DHANUVACHAPURAM
DEPARTMENT OF MATHEMATICS
QUESTION BANK
4TH SEMESTER MATHEMATICS CORE
MM1441:Elementary Number Theory Calculus II (2018 onwards)

[2 marks questions]

1. Prove that no prime of the form 4n+3 can be expressed as the sum of two squares
2. Determine whether 1928388 is divisible by 11
3. Determine whether 73215 is divisible by 9
4. Solve the congruence 12x$\equiv 48 (mod\ 18)$
5. Solve the congruence 12x$\equiv 6 (mod\ 7)$
6. Prove that if a$\equiv b (mod\ m)$ then $a^n \equiv b^n (mod\ m)$ for any positive integer n
7. Prove that the congruence relation is symmetric
8. State Euler's formula
9. State Fubini's theorem
10. Compute the least residue of $2^{340} (mod\ 341)$
11. Compute the least residue x such that $x^2 \equiv 1\ (mod\ 8)$
12. Using inverse find the incongruent solutions of the linear congruence 5x$\equiv 3(mod\ 6)$
13. Determine whether N=16,151,613,924 is a square.
14. Evaluate $\int_{-1}^{0}\int_{2}^{6} dxdy$
15. Evaluate $\int_{-1}^{0}\int_{2}^{6} dxdy$
16. Evaluate $\int_{4}^{3}\int_{2}^{4} 40 - 2xy$ dy dx
17. Evaluate $\int_{1}^{3}\int_{2}^{4} 40 - 2xy$ dy dx
18. Evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2 + y^2)^{\frac{3}{2}} dydx$
19. Evaluate $\int_{1}^{a}\int_{1}^{b} \frac{1}{xy}$ dxdy
20. Evaluate $\int_{-1}^{2}\int_{0}^{3}\int_{0}^{2} 12xy^2z^3\, dzdydx$
21. Evaluate $\iint_R y^2x$dA over the rectangle R= {(x,y):$-3 \le$x$\le 2,\ 0 \le$y≤ 1
22. Find $\iint_R y\ \ xdxdy$ where R is the triangular region with vertices (0,0),(2,0),(0,1).

23. Use a double integral to find the area of the region R enclosed between the parabola $y = \frac{x^2}{2}$ and the line y=2x
24. Find the volume of the solid enclosed by the surface $z = \frac{x}{y}$ and the rectangle $0 \leq x \leq 4$ and $0 \leq y \leq e^2$ in the xy-plane.
25. Find the surface area of the portion of the surface $z=\sqrt{4 - x^2}$ that lies above the rectangle R in the xy-plane where the coordinates satisfy $0 \leq x \leq 1$ and $0 \leq y \leq 4$
26. Write the converting formula for three dimensional cartesian to spherical and to cylindrical coordinates.
27. Explain Jacobian of transformation in 2 variables.
28. Find the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ were $x = r\cos\theta,\ y = r\sin\theta$
29. Use a polar double integral to find the area enclosed by the three petalled rose r = sin3θ
30. Find the divergence and curl of the vector field $F(x,y,z)= x^2y\,i+2y^3z\,j + 3z\,k$
31. Find the divergence and curl of the vector field $F(x,y,z)= x^2\,i+y^2\,j + z^2\,k$
32. Distinguish between del operator and Laplacian operator.
33. Evaluate the line integral $\int_C xy + z^3 dS$ from (1,0,0) to (-1,0,π) along the helix C that is represented by the parametric equation $x = \cos t,\ y = \sin t,\ z = t,\ o \leq t \leq \pi$
34. Evaluate $\int_C 2xydx + (x^2 + y^2)dy$ along the circular arc C given by x=cos t, y = sin t ($0 \leq t \leq \pi/2$)
35. Evaluate $\int_C F.dr$, where F(x,y)= cos x i + sin x j and C is the oriented curve C: r(t)= $\frac{-\pi}{2}i + t\,j,\ 1 \leq t \leq 2$
36. State Gauss's Law for inverse square fields
37. State Green's Theorem
38. Sketch the vector field $F(x, y) =- yi + xj$
39. Show that the vector field $F(x,y)=(ye^{xy} - 1)i + (xe^{xy})j$ is conservative. Also find the potential function $\phi(x, y)$.
40. Find the gradient field of $\phi(x, y)$.$=2x^2 + y$

[4 marks questions]

41. (a) If $a\equiv b(mod\, m)$ and $c\equiv d(mod\, m)$ then prove that $a-c\equiv b - d(mod\, m)$
    (b) If $a\equiv b(mod\, m)$ and c is any integer , then prove that $a+c\equiv b + c\ (mod\, m)$
42. Using the Pollard Rho method, factor the integer 3893
43. Using Pollard p-1 method, find the non trivial factor of n= 2813
44. Using Pollard rho method, with $x_0 = 2$and $f(x)= x^2 + 1$, find the canonical decomposition of 3893.
45. Solve the linear system $x\equiv 1(mod\ 3)$
    $x\equiv 3(mod\ 4)$

x≡ $4(mod\ 7)$

x≡ $7(mod\ 11)$

46. State and prove Wilson's Theorem
47. State and prove Fermat's Little theorem
48. Prove: a positive integer a is self invertible modulo p iff a≡$\pm\ 1(mod\ p)$
49. Prove that no integer of the form 8n+7 can be expressed as a sum of three squares.
50. Find the remainder when 18! is divided by 23.
51. Find the remainder when 16^{53} is divided by 7.
52. Evaluate $\int_{0}^{2}\int_{\frac{y}{z}}^{1} e^{x^2}dxdy$
53. Change the order of integration and hence evaluate $\int_{0}^{2}\int_{y/2}^{1} cos(x^2)dxdy$
54. Evaluate $\int_{0}^{a}\int_{0}^{a}\int_{0}^{a} xy + xz + yz\ dxdydz$
55. Evaluate $\int_{0}^{1a}\int_{y^2}^{1}\int_{0}^{1-x} xdz\ dydx$
56. Find the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ were u=xy, v=y, w= x-z
57. Find the Jacobian $\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)}$ for the spherical coordinates
58. Find the surface area of that portion of the paraboloid z=$x^2 + y^2$ below the plane z=1.
59. Find by double integration the area between the parabolas y=4x-x^2 and the line y=x
60. Use cylindrical coordinates to evaluate $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} x^2dzdydx$
61. Use cylindrical coordinates to find the volume of the solid enclosed by the paraboloid z= $x^2 + y^2$ and the plane z= 16.
62. Evaluate the triple integral $\iiint_G 13xy^2z^3\ dv$ over the rectangular box G defined by the inequalities $-1 \le x \le 2, 0 \le y \le 3, 0 \le z \le 2$
63. Use triple integral to find the volume of the solid in the first octant bounded by the coordinate planes and the plane 3x+6y+4z=12.
64. Use triple integral to find the volume of the solid bounded by the surface $y = x^2$ and the planes y+z=4 and z = 0..
65. Evaluate $\iint_R \frac{x-y}{x+y}dA$ where R is the region enclosed by $x - y = 0,\ x - y = 1,\ x + y = 1,\ x + y = 3$
66. Evaluate the surface integral $\iint_\sigma x^2dS$ over the sphere $x^2 + y^2 + z^2 = 1$

67. Evaluate $\int_C 2xydx + (x^2 + y^2)dy$ along the circular arc C given by $x = cost,\ y = sint,\ 0 \le t \le \pi/2$.
68. Find the work done by the force field F on a particle that moves along the curve F = xy i + x^3j ; C: x=y^2 $from\ (0, 0)\ to\ (1, 1)$.
69. Find the work done by the force field F(x,y) =$(e^x - y^3)i + (cosy + x^3)j$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counterclockwise direction.
70. Use the Divergence theorem to find the outward flux of the vector field F(x,y,z)= 2x i + 3y j + z^2k across the unit cube.
71. Using the parametrization evaluate the line integral $\int_C (1 + xy^2)ds\ ;\ C: r(t) = (1 - t)i + (2 - 2t)j\ ,\ 0 \le t \le 1$
72. Find $\int_C F.dr$ for $F(x, y) = (e^y + ye^x)i + (xe^y + e^x)j;\ \ C: r(t) = sin(\pi t/2)i + lnt\,j;\ 1 \le t \le 2.$
73. Find ∇.(FXG) if F(x,y,z)=2xi+j+5yk and G(x,y,z)=xi+yj-zk
74. Find $\nabla(\frac{x-y}{x+y})$
75. Show that the integral $\int_C yzdx + xz^2dy + yxdz$ is not independent of path.
76. Use Green's theorem to evaluate $\int_C x^2ydx + xdy$ along the triangular path (0,0),(1,0),(1,2).
77. Use Green's theorem to evaluate $\int_C (x^2 - 3y)dx + 3xdy$ along the circle $x^2 + y^2 = 4$

[15 marks questions]

78. (a) State Fermat's little Theorem and use it to find the remainder when $24^{1947}$is divided by 17
    (b) Using Chinese Remainder Theorem , solve the linear system of congruence x≡ $1(mod\ 3)$, x≡ $2(mod\ 5)$, x≡ $3(mod\ 7)$
79. (a) State and solve Mahavira's puzzle.
    (b) Find the remainder when 1!+2!+3!+.....+100! Is divided by 15
80. (a) State and prove Wilson's theorem
    (b) State and prove Chinese Remainder theorem
81. Evaluate the integral by reversing the order of integration: $\int_0^2\int_0^{\frac{x}{2}} e^x e^y dydx$; $\int_0^{\frac{\pi}{2}}\int_y^{\frac{\pi}{2}} \frac{sinx}{x} dxdy$
82. Find the area of the region bounded by $y = 2x^3,\ 2x + y = 4\ and\ the\ x\ axis.$

83. Convert to spherical coordinates and evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{1+x^2+y^2+z^2} dzdydx$

84. (a) Find the volume of the solid enclosed between the paraboloids z= $5x^2 + 5y^2$ $and\ z = 6 - 7x^2 - y^2$

(b) Use spherical coordinates to find the volume of the solid G bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below the cone z= $\sqrt{x^2 + y^2}$

85. (a) Find the surface area of that portion of the surface z= $\sqrt{4 - x^2}$ that lies above the rectangle R in the xy-plane whose coordinate satisfy $0 \le x \le 1, 0 \le y \le 4$

(b) Evaluate $\iint_R e^{xy} dA$ where R is the region enclosed by the lines $y = \frac{x}{2}$ $and\ y = x\ and\ the\ hyperbolas\ y = \frac{2}{x}\ and\ y = \frac{2}{x}$

86. (a) Using the Triple integral find the volume of the solid with the cylinder $x^2 + y^2 = 9$ and between the planes z=1 and x+z=5

(b) Use spherical coordinates to evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z^2\sqrt{x^2 + y^2 + z^2} dzdydx$

87. Suppose that a semi circular wire has the equation $y = \sqrt{25 - x^2}$ $and\ that\ its\ mass\ density\ is\ \delta(x, y) = 15 - y$. Find the mass of the wire.

88. Find the volume of the region G enclosed by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

89. Find the centroid of the solid G bounded below by the cone z= $\sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 9$

90. Evaluate the surface integral $\iint_R xzdS$ where σ is the part of the plane x+y+z=1 that lies in the first octant.

91. State Divergence theorem and verify it for F(x,y,z)= xy i + yz j + xz k , σ is the surface of the cube bounded by the planes x=0, x=2, y=0, y=2, z=0, z=2

92. (a)State Divergence theorem and verify it for the field F(x,y,z)= x i + y j + z k , over the sphere $x^2 + y^2 + z^2 = a^2$

(b) Suppose that a curved lamina $\sigma\ with\ constant$ density $\delta(x, y, z) = \delta_0$is the portion of the paraboloid $z = x^2 + y^2$below the plane z=1. Find the mass of the lamina.

93. State Stoke's theorem and verify it for F(x,y,z) = $x^2 i + y^2 j + z^2 k$ and σ is the portion of the cone z= $\sqrt{x^2 + y^2}$ $below\ the\ plane\ z = 1$

94. Verify Stokes theorem for the vector field $F(x, y, z) = 2zi + 3xj + 5yk$ taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \ge 0$ with upward orientation and C

to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane.

95. Show that the vector field F(x,y)=$2xy^3 i + (1 + 3x^2 y^2) j$ is conservative. Also find the potential function $\phi(x, y)$.
96. Given the vector field $F = yi - xj + zk$, verify Stokes Theorem for the hemispherical surface $x^2 + y^2 + z^2 = a^2$, z≥ 0.
97. $F = 2xzi + 2yz^2 j + (x^2 + 2y^2 z - 1)k$. Show that F is conservative vector field and hence find ϕ.
98. Verify Green's theorem for $\oint_C e^y dx + ye^x dy$ where(1) C:circle $x^2 + y^2 = 1$

    (2)Cis the boundary of the region enclosed by $y = x^2$ and $x = y^2$
99. Find the flux of $F(x, y, z) = xi + yj + 2zk$ through the portion of the paraboloid $z = 4 - x^2 - y^2$ that is on or above the xy-plane with upward orientation.
100. Use Stoke's theorem to evaluate $\iint_\sigma curlF.n\,ds$ where $F(x, y, z) = (z - y)i + (x + z)j - (x + y)k$ and σ is the portion of the paraboloid $z = 2 - x^2 - y^2$ on or above the plane z=1 with upward orientation.

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