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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2023

First Degree Programme under CBCSS

Mathematics

Core Course IX

MM 1641 : REAL ANALYSIS – II

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Evaluate $\lim\limits_{x \to 7/4} \dfrac{|x-2|}{x-2}$

2. State true or false: Every uniformly continuous function is continuous.

3. Determine the points of discontinuity of the greatest integer function.

4. State the mean value theorem.

5. Define a uniformly continuous function.

6. Define a differentiable function at a point.

P.T.O.

7. Give an example of a real valued function which is discontinuous at every point of R.

8. Define upper integral of a function f.

9. When do you say that a bounded real function f is integrable on [a,b]?

10. State true or false: If $|f|$ is integrable on [a,b] then f is also integrable on [a,b].

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. Evaluate $\lim_{x \to 0} \frac{x^2}{|x|}$

12. Prove that the Dirichlet's function f defined on R by $f(x) = \begin{cases} 1 \text{ if } x \text{ is irrrational} \\ -1 \text{ if } x \text{ is rational} \end{cases}$

is discontinuous at every point.

13. If f: $A \to$ R and g: $A \to R$ are continuous at a point $c \in A$, show that $f(x) + g(x)$ is also continuous at c.

14. Is the function $f(x) = \frac{1}{x}$ uniformly continuous on (0, 1]? Justify.

15. Prove that $\{f(x_n)\}$ is a Cauchy sequence for every Cauchy sequence $\{x_n\}$ in R where f is a uniformly continuous function.

16. If f is differentiable in (a, b) and $f'(x) \leq 0$ for all $x \in (a, b)$, show that f is monotonically decreasing.

17. Show by an example that a bounded function in [a,b] need not be continuous in [a,b].

18. If f: $A \to R$ is differentiable at a point $c \in A$, then f is continuous at c as well.

19. Find the value of δ for the function $f(x) = x^2 + 4x + 3$ to be uniformly continuous in the interval [-1,1], given $\varepsilon = \frac{1}{10}$.

20. Check whether the following function is integrable over [0,1]: $f(x) = 1$ if $x \in [0,1]$ and x is rational and $f(x) = 0$ if $x \in [0,1]$ and x is irrational.

21. Show that $\int_{\underline{a}}^{b} f\,dx \geq \int_{a}^{\bar{b}} f\,dx$.

22. Show that if f and g are bounded and integrable on [a,b], such that $f \geq g$, then $\int_{a}^{b} f\,dx \geq \int_{a}^{b} g\,dx$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Test the continuity of the function at x = 0 $f(x) = \begin{cases} x \sin\frac{1}{x}, for\ x \neq 0 \\ 0, for\ x = 0 \end{cases}$

24. Explain Lipschitz functions with the geometrical interpretation.

25. Show that a uniformly continuous function preserves Cauchy sequences.

26. Suppose f is a real differentiable function on [a,b] and suppose $f'(a) < \lambda < f'(b)$. Prove that there is a point $x \in (a,b)$ such that $f'(x) = \lambda$.

27. State and prove chain rule of differentiation.

28. State and prove Darboux's theorem.

29. Prove that, if f is monotonic in [a,b], it is integrable in [a,b].

30. If f and g are integrable in [a,b] then show that fg is also integrable in [a,b].

31. Show that the Dirichlet's function $g(x) = \begin{cases} 1 \, for \; x \; rational \\ 0 \, for \; x \; irrational \end{cases}$ is not integrable.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks.

32. Let f: $A \rightarrow R$ be continuous on A. If $K \subseteq A$ is compact, then prove that f (K) is compact as well.

33. State and prove Intermediate value theorem. Is the converse true? Justify your answer.

34. Prove that a bounded function f is integrable on [a,b] if and only if for every $\varepsilon > 0$ there exists a partition P such that U (P,f) – L (P, f) < ε.

35. If f is bounded and integrable on [a,b] and k is a number such that $|f(x)| \leq k$ for all x $\in [a, b]$. Prove that $\left| \int_a^b f \, dx \right| \leq k(b-a)$.

(2 × 15 = 30 Marks)