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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Physics

MM 1131.1 : Mathematics I — CALCULUS WITH APPLICATIONS IN PHYSICS — I

(2020 Admission)

Time : 3 Hours Max. Marks : 80

PART – I

Answer **all** questions. Each question carries **1** mark.

1. Find the derivative of $f(x) = x^3 \sin x$.

2. State Mean value theorem.

3. If a function $f(x)$ has a minimum $x = a$, then the second derivative $f''(x)$ at $x = a$ is ————.

4. The mean value m of a function between two limits a and b is defined by ————.

5. $\int \tan x dx =$ ————

P.T.O.

6. Find the sum $1^3 + 2^3 + ... + 100^3$.

7. Define conditional convergence of an infinite series.

8. Give a necessary condition for the convergence of a series of positive terms $\Sigma u_{n.}$

9. Let $v = i + 2j + 3k$. Find $3v$.

10. Define the vector product of two vectors a and b.

(10 × 1 = 10 Marks)

PART – II

Answer **any eight** questions. Each question carries **2** marks.

11. Find the derivative with respective to x of $f(x) = x^2(x^3 + 4)$.

12. Find the derivative with respect to x of $f(t) = 2at$, where $x = at^2$.

13. Using logarithmic differentiation find the derivative with respect to x of $y = a^x$.

14. Find the stationary points of the function $x^4 + 4x^3 - 2$.

15. Evaluate the integral $\int x^3 e^{-x^2} dx$.

16. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 5$.

17. Evaluate the integral $\int \ln x dx$.

18. Find the mean value of the function $f(x) = x^2$ between the limits $x = 2$ and $x = 4$.

19. Sum the integers between 1 and 1000 inclusive.

20. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n!+1}$ converges.

21. Check the convergence of the series $\sum_{n=1}^{\infty} n$.

22. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)}$.

23. Find the scalar triple product $a \cdot (b \times c)$ of the three vectors $a = -2i + 3j + k$, $b = 4j$ and $c = -i + 3j + 3k$.

24. Find the area of the parallelogram whose adjacent sides are given by the vectors $a = 3i + j + 4k$ and $b = i - j + k$.

25. Find the direction of the line of intersection of the two planes $x + 3y - z = 5$ and $2x - 2y + 4z = 3$.

26. Find the vector product of two vectors $a = 2i - 3j + k$ and $b = 4i - j + 5k$.

(8 × 2 = 16 Marks)

PART – III

Answer **any six** questions. Each question carries **4** marks.

27. Find $\frac{dy}{dx}$ for $x^2 + y^2 = 9$.

28. Find the fourth order derivative of the function $f(x) = \sinh x$.

29. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$.

30. Evaluate the integral $\int e^{ax} \cos bx \, dx$.

31. Evaluate $\int_1^{\infty} \frac{dx}{x^2 + 1}$.

32. Find the sum $\sum_{n=1}^{N} (n+1)(n+3)$.

33. Expand the function $\sin x$ as a Maclaurin series at $x = 0$.

34. State Leibnitz' theorem and find the n^{th} derivative of $y = x^3 e^{nx}$.

35. Describe alternating series test and $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

36. A point P divides a line segment AB in the ratio $\lambda : \mu$. If the position vectors of the points A and B are a and b, respectively, find the position vector of the point P.

37. Find the angle between the vectors $a = i + 2j + 3k$ and $b = 2i + 3j + 4k$.

38. Find the volume of the parallelepiped with sides $a = i + 2j + 3k$, $b = 4i + 5j + 6k$ and $c = 7i + 8j + 10k$.

(6 × 4 = 24 Marks)

PART – IV

Answer **any two** questions. Each question carries **15** marks.

39. (a) For the function $f(x) = 3x^3 + 9x^2 + 2$, determine the stationary points and their nature.

(b) Determine inequalities satisfied by $\ln x$ for suitable values of x.

40. (a) Find the area of the ellipse $\frac{1}{p^2} = \frac{\cos^2\phi}{a^2} + \frac{\sin^2\phi}{b^2}$ with semi-axes a and b.

(b) Show that the value of the integral $\int_0^1 \frac{1}{(1+x^2+x^3)^{1/2}}$ lies between 0.810 and 0.882.

41. (a) Find the volume of the solid generated by revolving the region bounded by $y = x^2$, the x-axis and $x = 2$ about y-axis.

(b) Calculate the length of the curve $y = \ln x$ from $x = \sqrt{3}$ to $x = \sqrt{15}$.

42. (a) Sum the series $1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots.$

(b) Determine the range of values of z for which the complex power series $1 - \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \ldots.$ converges.

43. (a) Find the minimum distance from the point P with coordinates (1, 2, 1) to the line $r = a + \lambda b$ where $a = i + j + k$ and $b = 2i - j + 3k$.

(b) The vertices of triangle ABC have position vectors a, b and c relative to some origin O. Find the position vector of the centroid G of the triangle.

44. Find the radius p of the circle that is the intersection of the plane $\hat{n} \cdot r = p$ and the sphere of radius a centred on the point with position vector c.

(2 × 15 = 30 Marks)
