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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, March 2020

First Degree Programme Under CBCSS

Complementary Course for Physics

MM 1431.1 MATHEMATICS IV (COMPLEX ANALYSIS, FOURIER SERIES AND FOURIER TRANSFORMS

(2014 – 2017 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. If $z_1 = 6 + 3i, z_2 = -2 + 3i$, what is the real part of $\frac{z_1}{z_2}$?

2. Using D'Moivre's Theorem, express $Cos\, 4\theta$ in terms of powers of $Cos\, \theta$ and $Sin\, \theta$.

3. Define analyticity of a complex function $f(z)$ at a point $z = z_0$ in a domain D in the complex plane.

4. Define harmonic functions.

5. State Cauchy's Integral theorem.

6. Obtain the singular points of the function $f(z) = Cot\, z$.

7. Find the residue of $f(z) = \frac{1}{(z^2+1)^3}$ at $z = i$.

P.T.O.

8. State Dirichlet conditions for the convergence of a Fourier series of a function $f(x)$ of period 2π.

9. Write the standard form of Fourier Cosine series and formulae for Fourier coefficients of the half range Cosine series of a function $f(x)$ in $(0, \pi)$.

10. If $F(s)$ is the Fourier transform of $f(x)$ then what is the Fourier transform of $f(ax)$ where $a \neq 0$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the questions **11** to **22**. These questions carry **2** marks each.

11. Find all distinct cube roots of i.

12. Show that an analytic function is constant if its modulus is constant.

13. Find an analytic function $f(z)$ whose real part is $u(x, y) = x^2 - y^2$.

14. Evaluate $\int_C \operatorname{Re} z\, dz$ where C is the shortest path form 1 + i to 3 + 2i.

15. Find the centre and radius of convergence of the power series $\sum_{n=1}^{\infty}\left(z + i\sqrt{2}\right)^n$.

16. Determine the location and nature of singularities of $f(z) = \frac{2}{z^3} - \frac{1}{z}$.

17. Find the residues of $f(z) = \frac{1}{\left(z^2 - 1\right)^2}$.

18. Expand $f(z) = \frac{1}{z^3 - z^4}$ as a Laurent's series that converges for $0 < |z| < 1$.

19. Evaluate $\int_{|z|=1} \frac{z}{(4z + i)^2} dz$.

20. Find the half range sine series of $f(x) = x, 0 < x < \pi$.

21. Find the Fourier series of periodicity 2 for $f(x)=\begin{cases}0, & -1<x<0\\ 1, & 0<x<1\end{cases}$.

22. Prove that Fourier transform is a linear operator.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the questions **23** to **31**. These questions carry **4** marks each.

23. Prove that the function $f(z)=|z|^2$ is differentiable at the origin but not analytic at the origin.

24. What are the values of $\int_C \frac{z+1}{z^2(z-2)}dz$ around the circles C where C is

(a) $|z|=1$ and

(b) $|z-2-i|=2$.

25. Expand $f(z)=\frac{1}{z(z^2-3z+2)}$ in the region $0<|z|<1$.

26. Obtain the residues of $f(z)=\frac{z^2-z+2}{(z+3i)(z-3i)(z+i)(z-i)}$ at its poles.

27. State Cauchy's Residue Theorem. Use Cauchy's Residue Theorem to evaluate the integral of the function $f(z)=\frac{z^5}{1-z^3}$ around the circle $|z|=2$, in the positive sense.

28. Show that $\int_{-\infty}^{\infty} \frac{Cos3x}{(x^2+1)^2}dx=\frac{2\pi}{e^3}$.

29. Obtain the Fourier series of the periodic function defined by $f(x)=\begin{cases}-\pi \text{ if } & -\pi<x<0 \\ x \text{ if } & 0<x<\pi\end{cases}$. Deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\infty$.

30. Expand $f(x)=x-x^2$ as a Fourier series in $-1<x<1$.

31. Find the Fourier transform of $f(x)=\begin{cases}1, & |x|<1 \\ 0, & |x|>1\end{cases}$, Hence evaluate $\int_0^\infty \frac{Sin\, x}{x}dx$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions from among the questions **32** to **35**. These questions carry **15** marks each.

32. (a) Show that the function $u(x,y)=x^3-3xy^2+3x^2-3y^2+1$ is harmonic and find the corresponding analytic function $f(z)$ in terms of z.

(b) If $f(z)=\sqrt{|xy|}$, check whether the Cauchy Riemann equations are satisfied at the origin. Is the function analytic at the origin? Justify your answer.

33. (a) Expand $\frac{1}{1-z^2}$ as a Taylor series about $z=i$.

(b) Evaluate $\int_C \frac{dz}{z^3(z+2)}$ where C is $|z|=3$.

34. Expand $f(x)=x^2, -\pi<x<\pi$ as a Fourier series of periodicity 2π. Hence deduce the following

(a) $\sum\frac{1}{n^2}=\frac{\pi^2}{6}$

(b) $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\ldots\ldots\infty$ and

(c) $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\infty$.

35. Find the Fourier transform of $e^{-a|x|}$, $a>0$, Deduce that $\int_{-\infty}^{\infty}\frac{Cos\, sx}{a^2+s^2}ds=\frac{\pi}{2a}e^{-ax}$, $x>0$.

(2 × 15 = 30 Marks)
