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Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, October 2019

First Degree Programme under CBCSS

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III — DIFFERENTIAL EQUATIONS, THEORY OF EQUATIONS AND THEORY OF MATRICES

(2014-2017 admissions)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. Each question carries **1** mark.

1. Solve $4xy\frac{dy}{dx} = y^2 - 1$.

2. Find the integrating factor of $\frac{1}{x}\frac{dy}{dx} + 4y = 2$.

3. Solve $4xy + 2x + (2x^2 + 3y^2)\frac{dy}{dx} = 0$.

4. Write the following system of equations as a matrix equation.
$2x + 6y + z = 7$
$x + 2y - z = -1$
$5x + 7y - 4z = 9$

5. Find the rank of the matrix $\begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$.

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6. Define a symmetric matrix.

7. Find the sum of the eigenvalues of the matrix $\begin{pmatrix} -2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

8. Solve $4x^3 + 16x^2 - 9x - 36 = 0$, the sum of two of the roots being zero.

9. Find the equation whose roots are $\frac{2}{3}, \frac{3}{2} \pm \sqrt{3}$.

10. State the Fundamental theorem of algebra.

SECTION – II

Answer **any eight** questions from among the questions 11 to 22. Each question carries **2** marks.

11. Solve the initial value problem $y - x = x\frac{dy}{dx}$, $y(1) = 2$.

12. Solve $y' = (x + y - 2)^2$.

13. Solve $\frac{dy}{dx} + xy = \frac{x}{y}$.

14. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$.

15. Solve :

$x + y = 1$
$4x - y = -6$
$2x - 3y = 8$

16. Find eigen values and eigenvectors of $A = \begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$.

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17. Diagonalize $A = \begin{pmatrix} -5 & 9 \\ -6 & 10 \end{pmatrix}$.

18. Obtain the characteristic polynomial of the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$.

19. Show that $x^5 - 2x^2 + 7 = 0$ has at least two imaginary roots.

20. Solve the equation $x^3 - 10x^2 + 8x + 64 = 0$, given that the product of two of the roots is the negative of the third.

21. Solve the equation $2x^3 + 3x^2 - 11x - 6 = 0$, given that the three roots form an Arithmetic sequence.

22. Solve $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$, given that one of the roots is $1 - \sqrt{5}$.

SECTION – III

Answer **any six** questions from among the questions 23 to 31. Each question carries **4** marks.

23. Find the orthogonal trajectories of the family of curves $(x - c)^2 + y^2 = c^2$.

24. Solve the initial value problem

$y'' - 6y + 25y = 0,\ y(0) = -3,\ y'(0) = -1.$

25. Solve $x^2y'' - xy' + y = \ln x$.

26. Solve

$x + 3y - 2z = -7$

$4x + y + 3z = 5$

$2x + 5y + 7z = 19$

27. Given that $\lambda = 1$ is a three-times repeated eigen value of the matrix $A = \begin{pmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$. Find the corresponding eigenvector.

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28. Diagonalize the matrix $A = \begin{pmatrix} 6 & -10 \\ 3 & -5 \end{pmatrix}$ and use the result to find A^{10}.

29. Find the nature of the roots of the equation $3x^4 + 12x^2 + 5x - 4 = 0$.

30. Solve the equation $x^5 - x^4 + 8x^2 - 9x - 15 = 0$, one root being $\sqrt{3}$ and another $1 - 2i$.

31. Using bisection method, find a real root of the equation $x^3 - 5x + 1 = 0$.

SECTION – IV

Answer **any two** questions from among the questions 32 to 35. Each question carries **15** marks.

32. (a) Solve $y''' - 2y'' - 4y' + 8y = 6xe^{2x}$.

(b) Solve $y''' - 4y'' - 5y' = 0$.

33. (a) Reduce the matrix $A = \begin{pmatrix} 4 & -2 & 2 & 3 & 1 \\ 2 & 0 & 0 & 3 & 2 \\ 4 & 1 & 2 & 5 & 1 \end{pmatrix}$ to its row-reduced echelon form and find its rank.

(b) Construct an orthogonal diagonalizing matrix for the symmetric matrix $A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$.

34. (a) Test for consistency and solve

$2x - 4y + 3z = 0$

$x + y - 2z = 0$

(b) Solve $(1+x)^2 y'' + (1+x) y' + y = 2\sin[\ln(1+x)]$.

35. Use the Newton-Raphson method to find the roots between 1 and 2, correct to six decimal places, of the equation $x^3 + 2x^2 - 5x + 1 = 0$.

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